Universal Gravitational Constant

EQUIPMENT

1	Gravitational Torsion Balance	AP-8215
1	X-Y Adjustable Diode Laser	OS-8526A
1	45 cm Steel Rod	ME-8736
1	Large Table Clamp	ME-9472
1	Meter Stick	SE-7333

INTRODUCTION

The Gravitational Torsion Balance reprises one of the great experiments in the history of physics—the measurement of the gravitational constant, as performed by Henry Cavendish in 1798.

The Gravitational Torsion Balance consists of two 38.3 gram masses suspended from a highly sensitive torsion ribbon and two1.5 kilogram masses that can be positioned as required. The Gravitational Torsion Balance is oriented so the force of gravity between the small balls and the earth is negated (the pendulum is nearly perfectly aligned vertically and horizontally). The large masses are brought near the smaller masses, and the gravitational force between the large and small masses is measured by observing the twist of the torsion ribbon.

An optical lever, produced by a laser light source and a mirror affixed to the torsion pendulum, is used to accurately measure the small twist of the ribbon.

THEORY

The gravitational attraction of all objects toward the Earth is obvious. The gravitational attraction of every object to every other object, however, is anything but obvious. Despite the lack of direct evidence for any such attraction between everyday objects, Isaac Newton was able to deduce his law of universal gravitation.

Newton's law of universal gravitation:

$$F = G\frac{m_1 m_2}{r^2}$$

where m_1 and m_2 are the masses of the objects, r is the distance between them, and $G = 6.67 \times 10^{-11}$ Nm^2/kg^2

However, in Newton's time, every measurable example of this gravitational force included the Earth as one of the masses. It was therefore impossible to measure the constant, G, without first knowing the mass of the Earth (or vice versa).

The answer to this problem came from Henry Cavendish in 1798, when he performed experiments with a torsion balance, measuring the gravitational attraction between relatively small objects in the laboratory. The value he determined for G allowed the mass and density of

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the Earth to be determined. Cavendish's experiment was so well constructed that it was a hundred years before more accurate measurements were made.

The gravitational attraction between a 15 gram mass and a 1.5 kg mass when their centers are separated by a distance of approximately 46.5 mm (a situation similar to that of the Gravitational Torsion Balance) is about 7×10^{-10} Newtons. If this doesn't seem like a small quantity to measure, consider that the weight of the small mass is more than two hundred million times this amount.

The enormous strength of the Earth's attraction for the small masses, in comparison with their attraction for the large masses, is what originally made the measurement of the gravitational constant such a difficult task. The torsion balance (invented by Charles Coulomb) provides a means of negating the otherwise overwhelming effects of the Earth's attraction in this experiment. It also provides a force delicate enough to counterbalance the tiny gravitational force that exists between the large and small masses. This force is provided by twisting a very thin beryllium copper ribbon.
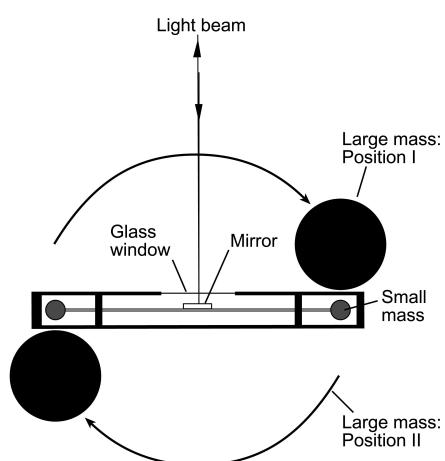

Figure 1: Top View

The large masses are first arranged in Position I, as shown in Figure 1, and the balance is allowed to come to equilibrium. The swivel support that holds the large masses is then rotated, so the large masses are moved to Position II, forcing the system into disequilibrium. The resulting oscillatory rotation of the system is then observed by watching the movement of the light spot on the scale, as the light beam is deflected by the mirror.

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SET UP

Figure 2: Removing a plate from the Chamber Box
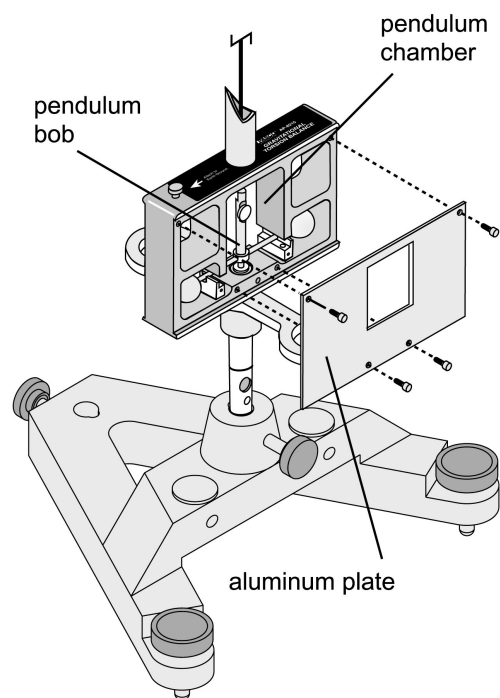
Preliminary Set Up
1. Place the support base on a flat, stable table that is located such that the Gravitational Torsion Balance will be at least 5 meters away from a wall or screen. For best results, use a very sturdy table, such as an optics table.
2. Carefully secure the Gravitational Torsion Balance in the base.
3. Remove the front plate by removing the thumbscrews.
4. Fasten the clear plastic plate to the case with the thumbscrews.

Leveling the Gravitational Torsion Balance

1. Release the pendulum from the locking mechanism by unscrewing the locking screws on the case, lowering the locking mechanisms to their lowest positions (Figure 3).
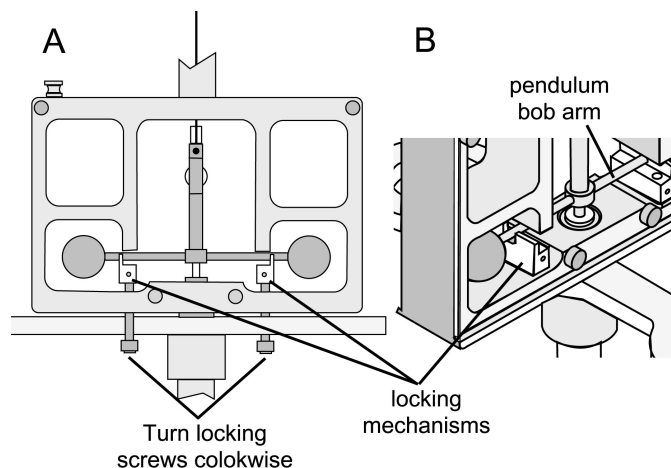

Figure 3: Lowering the Locking Mechanism to Release the Pendulum Bob Arms

2. Adjust the feet of the base until the pendulum is centered in the leveling sight (Figure 4). (The base of the pendulum will appear as a dark circle surrounded by a ring of light).
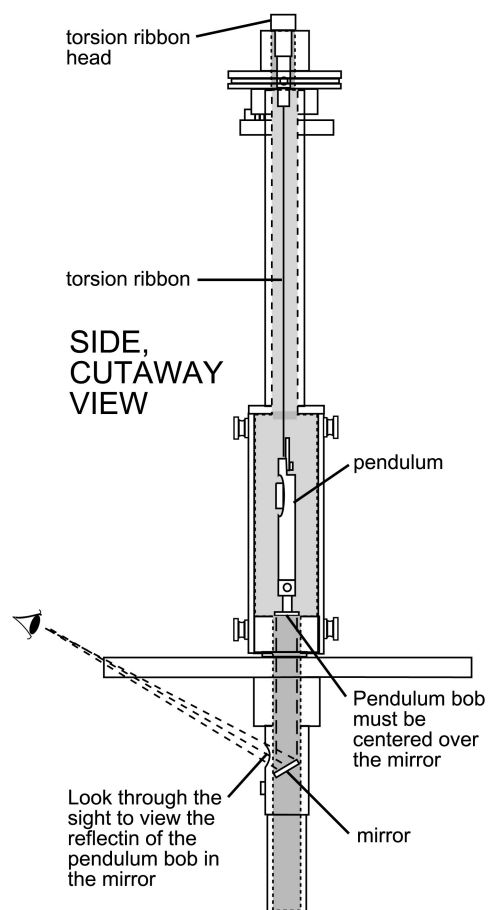
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3. Orient the Gravitational Torsion Balance so the mirror on the pendulum bob faces a screen or wall that is at least 5 meters away.

torsion ribbon head

torsion ribbon

SIDE, CUTAWAY VIEW

pendulum

Pendulum bob must be centered over the mirror

Look through the sight to view the reflectin of the pendulum bob in the mirror

mirror

Figure 4: Using the Leveling Sight

Ribbon Head

Philips Screw

Grasp the torsion ribbon head and loosen the Phillips screw.

A   B

The bottom of the pendulum bob should be flush with the floor of the chamber.
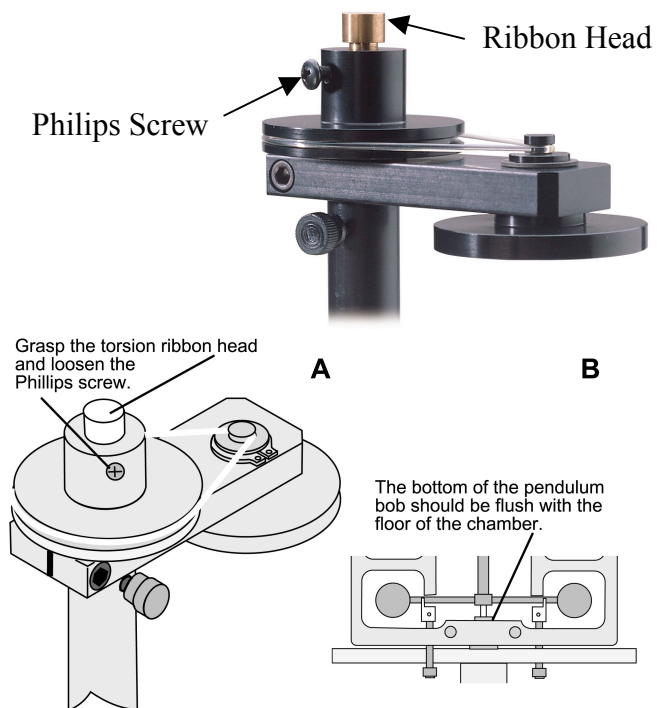
Figure 5: Adjusting the Height of the Pendulum

**Vertical Adjustment of the Pendulum**

The base of the pendulum should be flush with the floor of the pendulum chamber. If it is not, adjust the height of the pendulum:

1. Grasp the torsion ribbon head and loosen the Phillips retaining screw (Figure 5a).
2. Adjust the height of the pendulum by moving the torsion ribbon head up or down so the base of the pendulum is flush with the floor of the pendulum chamber (Figure 5b).
3. Tighten the retaining (Phillips head) screw.

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Rotational Alignment of the Pendulum Bob Arms (Zeroing)

TOP, CUTAWAY VIEW

The pendulum bob arm must be centered rotationally between the plates
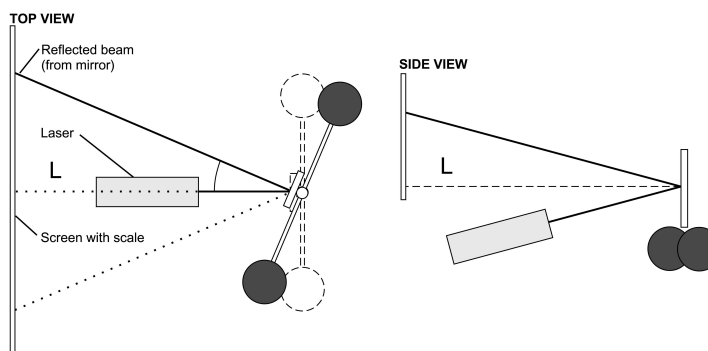
small mass

case plates
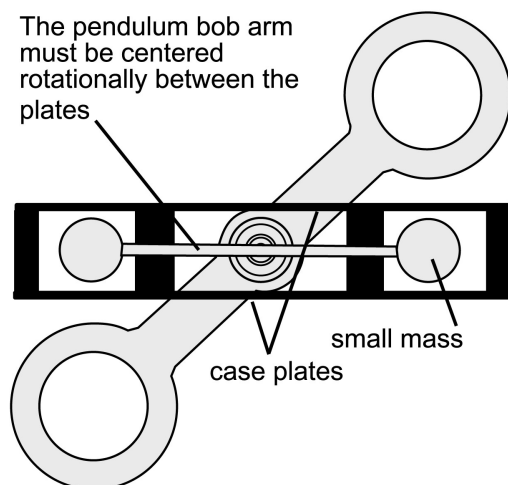
The pendulum bob arms must be centered rotationally in the case — that is, equidistant from each side of the case (Figure 6). To adjust them:

1. Mount a metric scale on the wall or other projection surface that is at least 5 meters away from the mirror of the pendulum.

2. Replace the plastic cover with the aluminum cover.

3. Set up the laser so it will reflect from the mirror to the projection surface where you will take your measurements (approximately 5 meters from the mirror). You will need to point the laser so that it is tilted upward toward the mirror and so the reflected beam projects onto the projection surface (Figure 7). There will also be a fainter beam projected off the surface of the glass window.
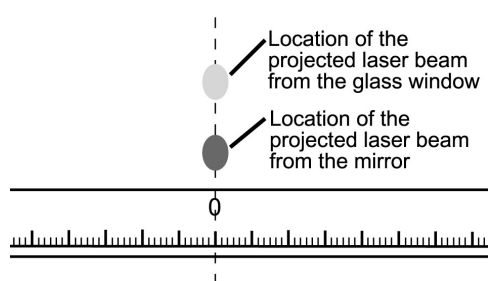
Figure 6: Aligning the Pendulum Bob Rotationally

TOP VIEW

Reflected beam (from mirror)

Laser

L

Screen with scale

SIDE VIEW

L

Figure 7a: Setting up the Optical Level (Illustrated View)

Figure 7b: Setting up the Optical Level

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Figure 8: Ideal Rotational Alignment

3. Rotationally align the case by rotating it until the laser beam projected from the glass window is centered on the metric scale (Figure 8).

4. Rotationally align the pendulum arm:
a. Raise the locking mechanisms by turning the locking screws until both of the locking mechanisms barely touch the pendulum arm. Maintain this position for a few moments until the oscillating energy of the pendulum is dampened.
b. Carefully lower the locking mechanisms slightly so the pendulum can swing freely. If necessary, repeat the dampening exercise to calm any wild oscillations of the pendulum bob.
c. Observe the laser beam reflected from the mirror. In the optimally aligned system, the equilibrium point of the oscillations of the beam reflected from the mirror will be vertically aligned below the beam reflected from the glass surface of the case (Figure 7).
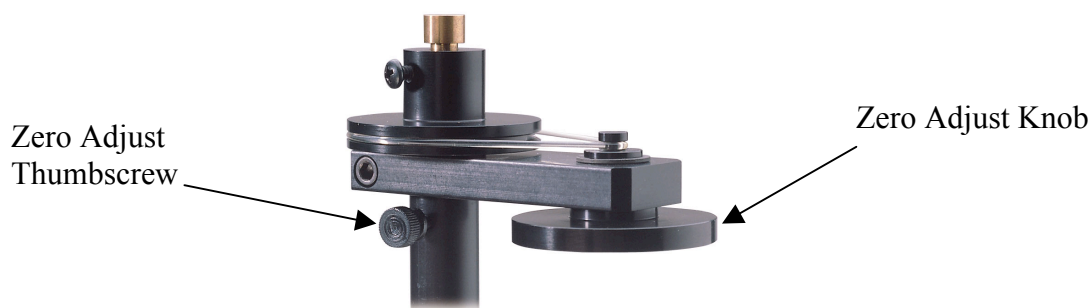

Figure 9: Refining the Rotational Alignment of the Pendulum Bob

d. If the spots on the projection surface (the laser beam reflections) are not aligned vertically, loosen the zero adjust thumbscrew, turn the zero adjust knob slightly to refine the rotational alignment of the pendulum bob arms (Figure 9), and wait until the movement of the pendulum stops or nearly stops.
e. Repeat steps 4a – 4c as necessary until the spots are aligned vertically on the projection surface.
5. When the rotational alignment is complete, carefully tighten the zero adjust thumbscrew, being careful to avoid jarring the system.

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Setting up for the Experiment

1. Take an accurate measurement of the distance from the mirror to the zero point on the scale on the projection surface (*L*) (Figure 7). (The distance from the mirror surface to the outside of the glass window is 11.4 mm.)
Note: Avoid jarring the apparatus during this setup procedure.

Figure 10: Attaching the Grounding Strap to the Grounding Screw

2. Attach copper wire to the grounding screw (Figure 10), and ground it to the earth.

Figure 11: Moving the Large Masses into Position 1
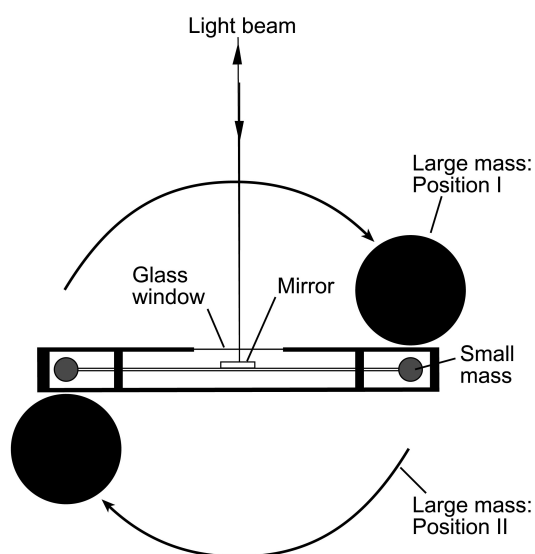
3. Place the large lead masses on the support arm, and rotate the arm to Position I (Figure 11), taking care to avoid bumping the case with the masses.
4. Allow the pendulum to come to resting equilibrium.
5. You are now ready to make a measurement using one of three methods: the final deflection method, the equilibrium method, or the acceleration method.
Note: The pendulum may require several hours to reach resting equilibrium. To shorten the time required, dampen the oscillation of the pendulum by smoothly raising the locking mechanisms up (by turning the locking screws) until they just touch the crossbar, holding for several seconds until the oscillations are dampened, and then carefully lowering the locking mechanisms slightly.

PROCEDURE

1. Once the steps for leveling, aligning, and setup have been completed (with the large masses in Position I), allow the pendulum to stop oscillating.
2. Turn on the laser and observe the Position I end point of the balance for several minutes to be sure the system is at equilibrium. Record the Position I end point (*S1*) as accurately as possible, and indicate any variation over time as part of your margin of error in the measurement.
3. Carefully rotate the swivel support so that the large masses are moved to Position II. The spheres should be just touching the case, but take care to avoid knocking the case and disturbing the system.

S_1 S_2

L

Position I

Position II

Intermediate
position
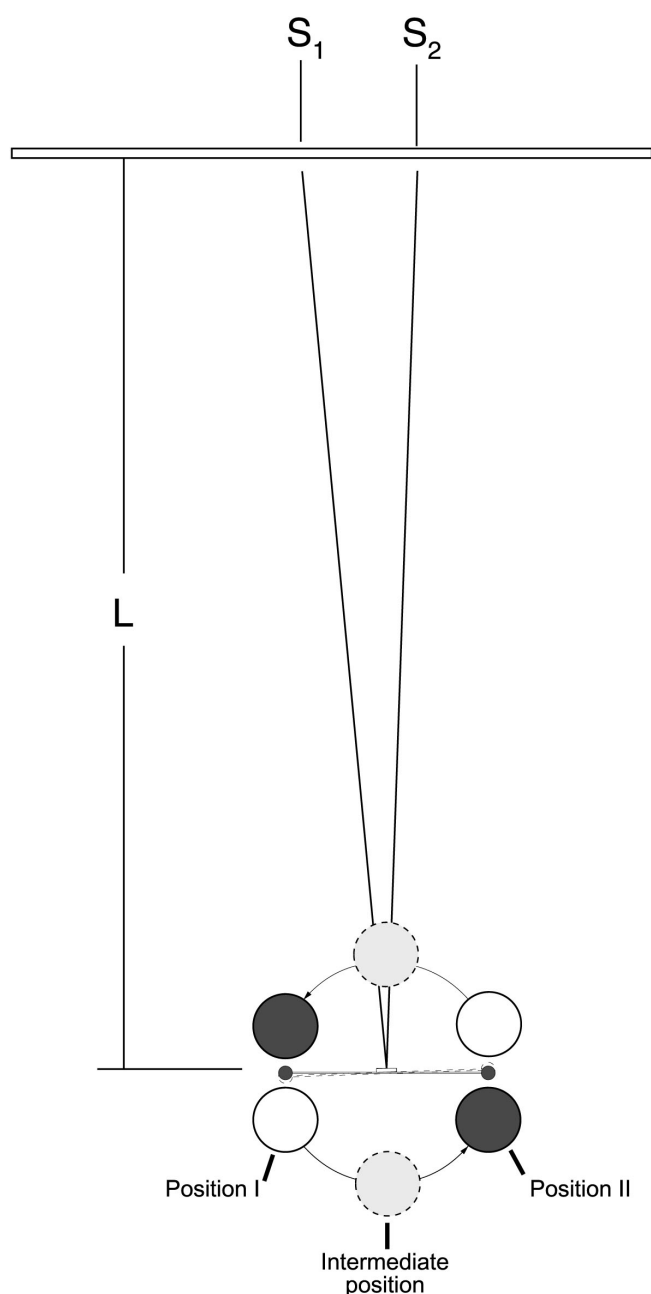
Note: You can reduce the amount of time the pendulum requires to move to equilibrium by moving the large masses in a two-step process: first move the large masses and support to an intermediate position that is in the midpoint of the total arc (Figure 12), and wait until the light beam has moved as far as it will go in the period; then move the sphere across the second half of the arc until the large mass support just touches the case. Use a slow, smooth motion, and avoid hitting the case when moving the mass support.

4. Immediately after rotating the swivel support to Position II, observe the light spot. Record the position of the light spot (S) and the time (t) every 15 seconds. Continue recording the position and time for about 45 minutes.

5. Rotate the swivel support to Position I. Repeat the procedure described in step 4.

Note: Although it is not imperative that step 5 be performed immediately after step 4, it is a good idea to proceed with it as soon as possible in order to minimize the risk that the system will be disturbed between the two measurements. Waiting more than a day to perform step 5 is not advised.

Figure 12: Two-step process of moving the large masses to reduce the time required to stop oscillating

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ANALYSIS

1. Construct a graph of light spot position versus time for both Position I and Position II. You will now have a graph similar to Figure 13.
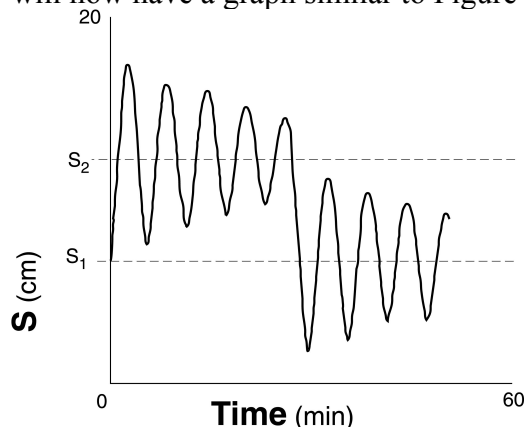

Figure 13: Typical Pendulum Oscillation Pattern Showing Equilibrium Positions

2. Find the equilibrium point for each configuration by analyzing the corresponding graphs using graphical analysis to extrapolate the resting equilibrium points S1 and S2 (the equilibrium point will be the center line about which the oscillation occurs). Find the difference between the two equilibrium positions and record the result as ΔS.

3. Determine the period of the oscillations of the small mass system by analyzing the two graphs. Each graph will produce a slightly different result. Average these results and record the answer as T.

4. Use your results and equation 1.9 below to determine the value of G.

Calculating the Value of G

Figure 14: Origin of Variables b and d
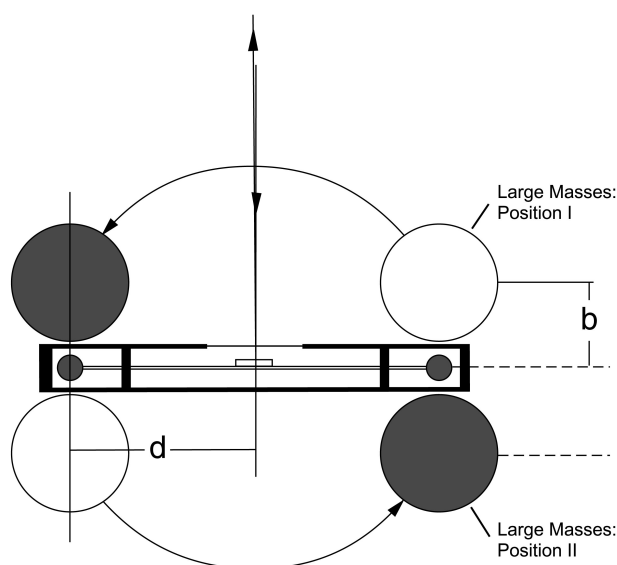
With the large masses in Position I (Figure 14), the gravitational attraction, F, between each small mass (m_2) and its neighboring large mass (m_1) is given by the law of universal gravitation:

$$F = \frac{Gm_1m_2}{b^2} \qquad\qquad (1.1)$$

where b = the distance between the centers of the two masses.

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The gravitational attraction between the two small masses and their neighboring large masses produces a net torque ($\boldsymbol{\tau}_{grav}$) on the system:

$$\boldsymbol{\tau}_{grav} = 2Fd \qquad (1.2)$$

where d is the length of the lever arm of the pendulum bob crosspiece.

Since the system is in equilibrium, the twisted torsion band must be supplying an equal and opposite torque. This torque ($\boldsymbol{\tau}_{band}$) is equal to the torsion constant for the band (κ) times the angle through which it is twisted (θ), or:

$$\boldsymbol{\tau}_{band} = -\kappa\theta. \qquad (1.3)$$

Combining equations 1.1, 1.2, and 1.3, and taking into account that $\boldsymbol{\tau}_{grav} = -\boldsymbol{\tau}_{band}$, gives:

$$\kappa\Theta = \frac{2dGm_1m_2}{b^2}$$

Rearranging this equation gives an expression for G:

$$G = \frac{\kappa\Theta b^2}{2dm_1m_2} \qquad (1.4)$$

To determine the values of θ and κ — the only unknowns in equation 1.4 — it is necessary to observe the oscillations of the small mass system when the equilibrium is disturbed. To disturb the equilibrium (from S1), the swivel support is rotated so the large masses are moved to Position II. The system will then oscillate until it finally slows down and comes to rest at a new equilibrium position (S2) (Figure 15).
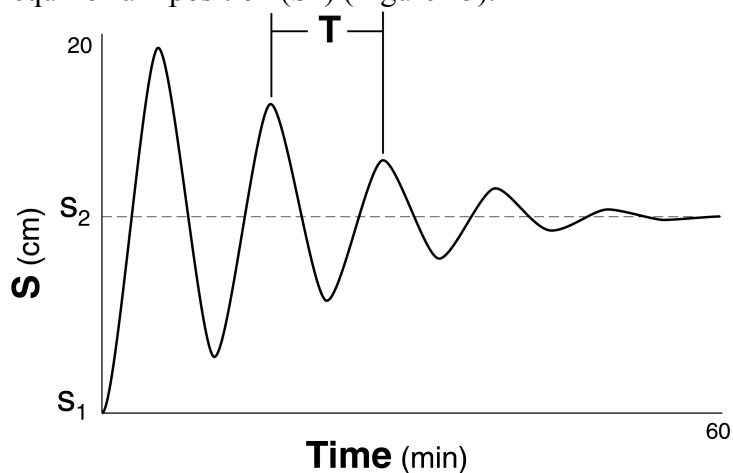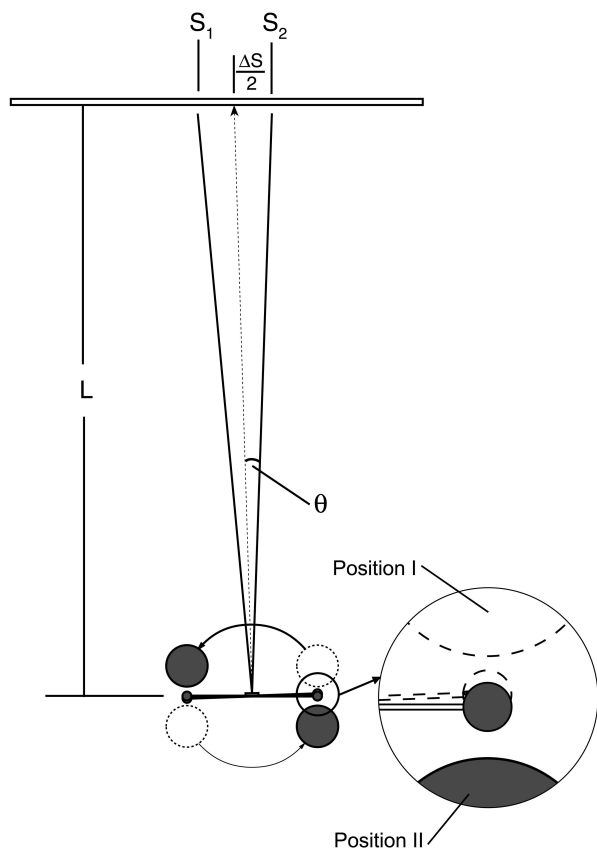

Figure 15: Graph of Small Mass Oscillations

At the new equilibrium position *S2*, the torsion wire will still be twisted through an angle θ, but in the opposite direction of its twist in Position I, so the total change in angle is equal to 2θ. Taking into account that the angle is also doubled upon reflection from the mirror (Figure 16):

$$\Delta S = S_2 - S_1$$

$$4\Theta = \frac{\Delta S}{L} \quad \text{or}$$

$$\Theta = \frac{\Delta S}{4L} \quad (1.5)$$

Figure 16: Diagram of the Experiment Showing the Optical Level

The torsion constant can be determined by observing the period (*T*) of the oscillations, and then using the equation:

$$T^2 = \frac{4\pi^2 I}{\kappa} \quad (1.6)$$

where *I* is the moment of inertia of the small mass system. The moment of inertia for the mirror and support system for the small masses is negligibly small compared to that of the masses themselves, so the total inertia can be expressed as:

$$I = 2m_2\left(d^2 + \frac{2}{5}r^2\right) \quad (1.7)$$

Therefore:

$$\kappa = 8\pi^2 m_2 \frac{d^2 + \frac{2}{5}r^2}{T^2} \quad (1.8)$$

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Substituting equations 1.5 and 1.8 into equation 1.4 gives:

$$G = \pi^2 \Delta S b^2 \frac{\left(d^2 + \frac{2}{5}r^2\right)}{T^2 m_1 L d} \qquad (1.9)$$

All the variables on the right side of equation 1.9 are known or measurable:

r = 9.55 mm
d = 50 mm
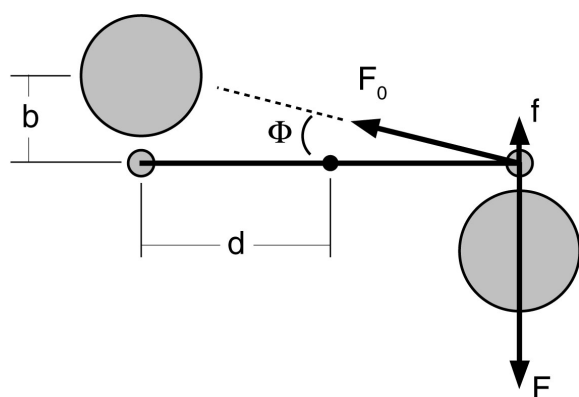b = 46.5 mm
m_1 = 1.5 kg
L = (Measure as in step 1 of the setup.)

By measuring the total deflection of the light spot (ΔS) and the period of oscillation (T), the value of G can therefore be determined.

5. The value calculated in step 4 is subject to the following systematic error. The small sphere is attracted not only to its neighboring large sphere, but also to the more distant large sphere, though with a much smaller force. The geometry for this second force is shown in Figure 17 (the vector arrows shown are not proportional to the actual forces).

Figure 17: Correcting the Measured Value of G

From Figure 17,

$$f = F_0 \sin \Phi$$
$$\sin \Phi = \frac{b}{\left(b^2 + 4d^2\right)^{1/2}}$$

The force, F_0 is given by the gravitational law, which translates, in this case, to:

$$F_0 = \frac{G m_2 m_1}{\left(b^2 + 4d^2\right)}$$

and has a component f that is opposite to the direction of the force F:

$$f = \frac{G m_2 m_1 b}{\left(b^2 + 4d^2\right)\left(b^2 + 4d^2\right)^{1/2}} = \beta F$$

This equation defines a dimensionless parameter, *b*, that is equal to the ratio of the magnitude of *f* to that of *F*. Using the equation

$$F = \frac{Gm_2 m_1}{b^2}$$

it can be determined that:

$$b = \frac{b^3}{\left(b^2 + 4d^2\right)^{3/2}}$$

From Figure 17,

$$F_{net} = F - f = F - bF = F(1 - b)$$

where F_{net} is the value of the force acting on each small sphere from *both* large masses, and *F* is the force of attraction to the nearest large mass only.
Similarly,

$$G = G_0 (1 - b)$$

where *G* is your experimentally determined value for the gravitational constant, and G_0 is corrected to account for the systematic error.
Finally,

$$G_0 = \frac{G}{(1 - b)}$$

Use this equation with equation 1.9 to adjust your measured value.

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