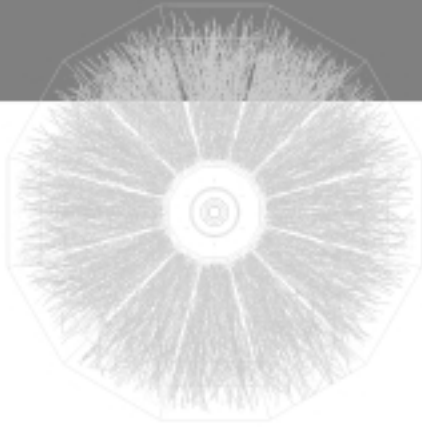




# More

## Derivation of Compton's Equation

Let  $\lambda_1$  and  $\lambda_2$  be the wavelengths of the incident and scattered x rays, respectively, as shown in Figure 3-21. The corresponding momenta are

$$p_1 = \frac{E_1}{c} = \frac{hf_1}{c} = \frac{h}{\lambda_1}$$

and

$$p_2 = \frac{E_2}{c} = \frac{h}{\lambda_2}$$

using  $f\lambda = c$ . Since Compton used the  $K_\alpha$  line of molybdenum ( $\lambda = 0.0711$  nm; see Figure 3-18b), the energy of the incident x ray (17.4 keV) is much greater than the binding energy of the valence electrons in the carbon scattering block (about 11 eV); therefore, the carbon electron can be considered to be free.

Conservation of momentum gives

$$\mathbf{p}_1 = \mathbf{p}_2 + \mathbf{p}_e$$

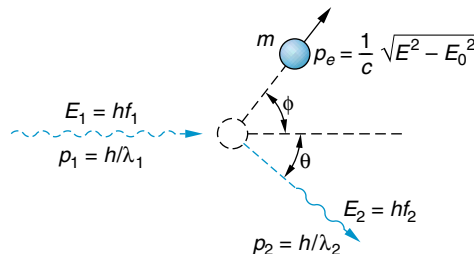


**Fig. 3-21** The scattering of x rays can be treated as a collision of a photon of initial momentum  $h/\lambda_1$  and a free electron. Using conservation of momentum and energy, the momentum of the scattered photon  $h/\lambda_2$  can be related to the initial momentum, the electron mass, and the scattering angle. The resulting Compton equation for the change in the wavelength of the x ray is Equation 3-40.

(Continued)

or

$$\begin{aligned} p_e^2 &= p_1^2 + p_2^2 - 2\mathbf{p}_1 \cdot \mathbf{p}_2 \\ &= p_1^2 + p_2^2 - 2p_1p_2 \cos \theta \end{aligned} \quad \mathbf{3-41}$$

where  $p_e$  is the momentum of the electron after the collision and  $\theta$  is the scattering angle for the photon, measured as shown in Figure 3-21. The energy of the electron before the collision is simply its rest energy  $E_0 = mc^2$  (see Chapter 2). After the collision, the energy of the electron is  $(E_0^2 + p_e^2c^2)^{1/2}$ .

Conservation of energy gives

$$p_1c + E_0 = p_2c + (E_0^2 + p_e^2c^2)^{1/2}$$

Transposing the term  $p_2c$  and squaring, we obtain

$$E_0^2 + c^2(p_1 - p_2)^2 + 2cE_0(p_1 - p_2) = E_0^2 + p_e^2c^2$$

or

$$p_e^2 = p_1^2 + p_2^2 - 2p_1p_2 + \frac{2E_0(p_1 - p_2)}{c} \quad \mathbf{3-42}$$

If we eliminate  $p_e^2$  from Equations 3-41 and 3-42, we obtain

$$\frac{E_0(p_1 - p_2)}{c} = p_1p_2(1 - \cos \theta)$$

Multiplying each term by  $hc/p_1p_2E_0$  and using  $\lambda = h/p$ , we obtain *Compton's equation*:

$$\lambda_2 - \lambda_1 = \frac{hc}{E_0} (1 - \cos \theta) = \frac{hc}{mc^2} (1 - \cos \theta)$$

or

$$\lambda_2 - \lambda_1 = \frac{h}{mc} (1 - \cos \theta) \quad \mathbf{3-40}$$