

2006-08-28 Problem Solutions

Problem 1: Tipler 1-3

The solution is obtained from Eq. (1-12), which is in the supplement entitled *The Michelson-Morley Experiment*, obtainable from the web site for the course. The equation says

$$\Delta N = \frac{2L}{\lambda} \frac{v^2}{c^2}, \quad (1)$$

where ΔN is the number of fringe shifts caused by rotating the interferometer, L is the length of each arm of the interferometer, λ is the wavelength of the light, and v is the velocity of Earth through the ether. On page 12 the book says that the orbital speed of Earth is $v = 30 \times 10^3$ m/s and that produces a fringe shift $\Delta N = 0.4$. To increase the fringe shift to $\Delta N = 1$, which is an increase of $1/0.4 = 2.5$, the square of the velocity v^2 must increase by the same factor of 2.5. Therefore the velocity must increase to $\sqrt{2.5}(30 \text{ km/s}) = 47.4 \text{ km/s}$.

Problem 2: Tipler 1-4

a) This problem is exactly like Example 1-3 with airplanes instead of boats. We use Eq. (1-8) on page 12, with $L = 12.5$ miles, $c = 130$ miles/hr, and $v = 20$ miles/hr. To maintain the analogy to Example 1-3, we chose the airplane flying perpendicular to the wind to be #1, and the plane flying parallel to the wind to be #2. Then #1 will win the race by a time

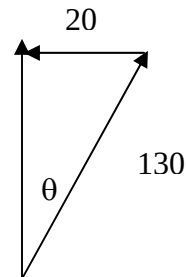
$$\Delta t = \frac{Lv^2}{c^3} = \frac{(12.5 \text{ miles})(20 \text{ miles/hr})^2}{(130 \text{ miles/hr})^3} = 0.0023 \text{ hours} = 8.2 \text{ s}. \quad (2)$$

The formula in Eq. (2) is approximate because its derivation used the binomial expansion and assumed that the ratio $(v/c)^2$ is small. In this problem $(v/c)^2 = 0.024$, so the answer is accurate to about 2 percent.

b) Plane #1 must head at an angle

$$\theta = \arcsin\left(\frac{20}{130}\right) = 8.8^\circ \quad (3)$$

into the wind.



Problem 3: Tipler 1-7

(This solution is from the authors' solution manual.)

$$v^2 = \frac{\Delta N \lambda c^2}{2L} = (0.01 \text{ fringe}) (590 \times 10^{-9} \text{ m}) (3.00 \times 10^8 \text{ m/s})^2 2(11 \text{ m})$$

$$v = 4.91 \times 10^3 \text{ m/s} \approx 5 \text{ km/s}$$

Problem 4: Tipler 1-20

This is an exercise in the binomial expansion (see the note about this formula available on the web site for the course),

$$(1+x)^p = 1 + px + \frac{1}{2} p(p-1)x^2 + \dots \quad (4)$$

This sum is usually an infinite series, but typically when we use this formula in physics the quantity x is very small, so higher powers of it are very very small and can be neglected. In this problem the quantity x is v^2/c^2 , and it is assumed to be small, so we terminate Eq. (4) after the second term. Then for part (a), the exponent is $p = -1/2$, and for part (b) it is $p = +1/2$.

$$\begin{aligned} \text{b) } \frac{1}{\gamma} &\equiv \sqrt{1 - \frac{v^2}{c^2}} = \left(1 - \frac{v^2}{c^2}\right)^{1/2} = 1 - \frac{1}{2} \frac{v^2}{c^2} + \dots, \\ \text{a) } \gamma &= \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = 1 + \frac{1}{2} \frac{v^2}{c^2}. \end{aligned} \quad (5)$$

From the formulas in Eq. (5), we get

$$\gamma - 1 = \frac{1}{2} \frac{v^2}{c^2} + \dots \quad \text{and} \quad 1 - \frac{1}{\gamma} = \frac{1}{2} \frac{v^2}{c^2} + \dots \quad \text{and so } \gamma - 1 \approx 1 - \frac{1}{\gamma} \quad . \quad (6)$$

Problem 5: Tipler 1-44

The problem gives a formula for the putative different length of the interferometer arm due to its motion through the ether: $L = L_p \sqrt{1 - \left(\frac{v}{c}\right)^2} = \frac{L}{\gamma}$. (The subscript p stands for “proper”, and it means the length of the rod at rest.) The *change* in the length is

$$\Delta L = L_p - L = L_p - \frac{L_p}{\gamma} = L_p \left(1 - \frac{1}{\gamma}\right) = L_p \left(\frac{1}{2} \frac{v^2}{c^2}\right). \quad (7)$$

The last step used the formula for $1 - 1/\gamma$ that was derived in Problem 1-20 and is valid for $v \ll c$. Substitute the numbers for Michelson’s interferometer (page 12 of book): $L = 11 \text{ m}$, $v = 3.0 \times 10^4 \text{ m/s}$, and get $\Delta L = 5.5 \times 10^{-8} \text{ m}$. Using the estimate for one atomic diameter given in the problem, this “shrinkage” comes out to be $5.5 \times 10^{-8} \text{ m} / (10^{-10} \text{ m/atomic diameter}) = 550 \text{ atomic diameters}$.