

2006-09-04 Problem Solutions

Tipler 1-18

In frame S' , the light beam has velocity components $u_x' = 0$, $u_y' = c$, that is the light is going parallel to the y' axis. Apply the “inverse” velocity transformation of Eq. (1-25) to this problem and get

$$u_x = \frac{0 + v}{1 + \frac{v}{c^2} \cdot 0} = v \quad (\text{This is obvious without a formula, right?}) \quad u_y = \frac{c}{\gamma \left(1 + \frac{v}{c^2} \cdot 0 \right)} = \frac{c}{\gamma}. \quad (1)$$

b) Apply the Pythagorean theorem to get the magnitude of the velocity in S .

$$\text{speed} = \sqrt{u_x^2 + u_y^2} = \sqrt{v^2 + \frac{c^2}{\gamma^2}} = \sqrt{v^2 + c^2 \left(1 - \frac{v^2}{c^2} \right)} = \sqrt{v^2 + c^2 - v^2} = c. \quad (2)$$

This is, of course, what it had to be.

Tipler 1-19

This problem is two successive applications of the “inverse” velocity transformation formula for velocity components along the x , x' and x'' axes.

a) Here there is a particle moving along the x'' -axis with speed $u_x'' = 0.9c$. The S'' frame is moving at speed $v_2 = 0.9c$ with respect to the S' frame. From the “inverse” velocity transformation for x -components, the velocity of the particle in S' is

$$u_x' = \frac{0.9c + 0.9c}{1 + \frac{(0.9c)(0.9c)}{c^2}} = \frac{1.8c}{1 + 0.81} = 0.9945c. \quad (3)$$

b) Now we apply the same formula again to go from frame S' to frame S .

$$u_x = \frac{0.9945c + 0.9c}{1 + \frac{(0.9945c)(0.9c)}{c^2}} = \dots (\text{some arithmetic omitted}) = 0.997c. \quad (4)$$

Tipler 1-43

$$(a) \gamma = 1/\sqrt{1-\beta^2} = 1/\sqrt{1-(0.92)^2} = 2.55$$

$$(b) \tau = 2.6 \times 10^{-8} s \quad \Delta t(lab) = \gamma \tau = 2.55(2.6 \times 10^{-8} s) = 6.63 \times 10^{-8} s$$

$$(c) N(t) = N_0 e^{-t/\tau} \quad (\text{Equation 1-31}) \quad L = \sqrt{1-\beta^2} L_0 = \sqrt{1-(0.92)^2} (50 m) = 19.6 m$$

Where L is the distance in the pion system. At $0.92c$, the time to cover $19.6m$ is:

$$t = 19.6m/0.92c = 7.10 \times 10^{-8} s. \text{ So for } N_0 = 50,000 \text{ pions initially, at the end of } 50m \text{ in the}$$

$$\text{lab, } N = (5.0 \times 10^4) e^{-7.1/2.6} = 3,260$$

$$(d) \text{ Immediately before covering } 50m \text{ at } 0.92c \text{ is } 1.81 \times 10^{-7} s \text{ and } N \text{ would be } N = (5.0 \times 10^4) e^{-18.1/2.6} = 47$$

This is what (d) says:

If there were no time dilation, the muons would take time

$$t = \frac{50 \text{ m}}{(0.92)(3 \times 10^8 \text{ m/s})} = 1.81 \times 10^{-7} \text{ s to travel the length of the tube. Some of them}$$

would decay along the way. Of the 50,000 that started the trip, the number remaining at the end would be

$$N(t) = N_0 \exp\left(-\frac{t}{\tau}\right) = (50,000) \exp\left(-\frac{1.81 \times 10^{-7} \text{ s}}{2.6 \times 10^{-8} \text{ s}}\right) = 47.$$

This is much smaller than the number that actually arrives. The experiment agrees with the result when time dilation is taken into account. So this experiment supports the validity of relativity theory.

Tipler 1-52

In the lab the proton has a velocity $u = 0.85c$ at an angle of 50° with the x -axis. The components of this velocity are

$$u_x = (0.85c) \cos 50^\circ, \quad u_y = (0.85c) \sin 50^\circ. \quad (5)$$

We transform these into the frame S' , which is moving at velocity $v = 0.72c$ (i.e. $\beta = 0.72$). First, we calculate that

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-0.72^2}} = 1.441. \quad (6)$$

Then the “forward” velocity transformation equations give

$$\begin{aligned}
u'_x &= \frac{u_x - v}{1 - \frac{v}{c^2} u_x} = \frac{(0.85c) \cos 50^\circ - 0.72c}{1 - \frac{(0.72c)[(0.85c) \cos 50^\circ]}{c^2}} = \frac{-0.1736c}{1 - 0.3934} = -0.286c, \\
u'_y &= \frac{u_y}{\gamma \left(1 - \frac{v}{c^2} u_x\right)} = \frac{(0.85c) \sin 50^\circ}{1.442 \left\{1 - \frac{(0.72c)[(0.85c) \cos 50^\circ]}{c^2}\right\}} = 0.745c.
\end{aligned} \tag{7}$$

The magnitude and direction of the velocity in frame S' are

$$u' = \sqrt{u'^2_x + u'^2_y} = 0.798c, \quad \theta' = \arctan\left(\frac{u'_y}{u'_x}\right) = \arctan\left(\frac{0.745}{-0.286}\right) = 111^\circ. \tag{8}$$

The angle is measured from the x' axis.

When you put these numbers into your calculator, it will probably tell you that the angle is -69° (that's what my calculator said). However, a vector at that angle has a negative y component and a positive x -component, and we already know that the signs are the other way around. So we have to add 180° to the calculator angle to get an angle that has the same tangent and has the correct signs for the components.