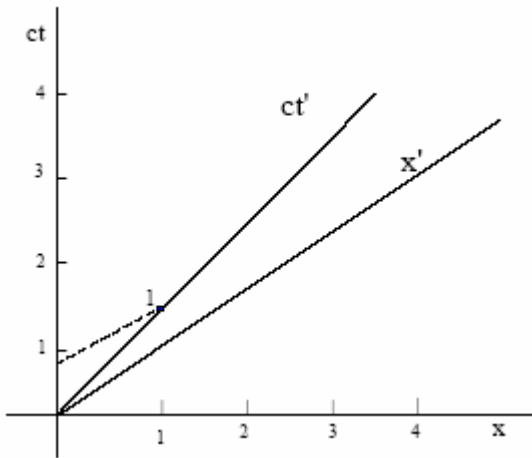


2006-09-06 Problem Solutions

Problem 1: Tipler 1-17

Here is the answer from the solution manual for the book.

(a) As seen from the diagram, when the observer in the rocket (S') system sees 1 c·s (light seconds) tick by on the rocket's clock, only 0.6 c·s have ticked by on the laboratory clock.



(b) When 10 seconds have passed on the rocket's clock, only 6 seconds have passed on the laboratory clock.

Let's elaborate on part (a).

The solution above took 1 c·s for the time interval just as an example; that wasn't given in the problem.

For this problem $\beta = 0.8$, $1/\beta = 1.25$ and $\gamma = 5/3 = 1.667$. This picture is constructed like Fig. 1-22 in the book, except it has a different value of β . The ct' axis has slope $1/\beta = 1.25$, and its equation is $ct = 1.25x$. It is also the line $x' = 0$. The x' axis has slope $\beta = 0.8$, so its equation is $ct = 0.8x$. It is also the line $ct' = 0$.

The point at the right-hand end of the dotted line has $ct' = c(1 \text{ s}) = 1 \text{ c} \cdot \text{s}$. The line which goes through that point and is parallel to the x' axis (which is the line $ct' = 0$) is the line $ct' = 1 \text{ c} \cdot \text{s}$. We get its equation from the Lorentz transformation equation

$$t' = \gamma \left(t - \frac{vx}{c^2} \right) \quad \text{or} \quad ct' = \gamma (ct - \beta x)$$

by setting $ct' = 1 \text{ c} \cdot \text{s}$ and solving for ct .

$$1 \text{ c} \cdot \text{s} = ct' = \gamma(ct - \beta x) = \frac{5}{3}(ct - 0.8x) \quad \text{or} \quad ct = 0.8x + \frac{3}{5}(1 \text{ c} \cdot \text{s}).$$

To get the value of t at $x = 0$ (the left-hand end of the dotted line on the figure), set $x = 0$ in the previous equation and solve for t . The c 's cancel and the result is

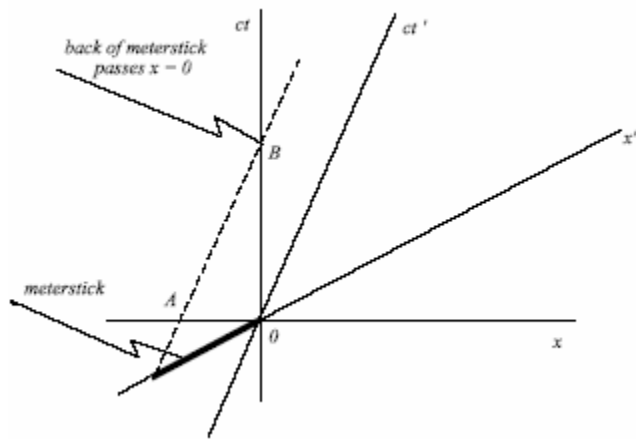
$$t = \frac{3}{5} \text{ s} = 0.6 \text{ s}.$$

The point is that the lab clock fixed at the origin ticks slower than the clocks on the rocket ship moving by. Using the picture to solve the problem is interesting but is more work than using the Lorentz transformation.

Part (b) is obtained by simple proportion from the answer to (a).

Problem 2: Tipler 1-23

- This is a simple problem in length contraction. For $v = 0.6c$, $\beta = 0.6$, $\gamma = [1 - (.6)^2]^{-1/2} = 1.25$. The contracted length of the rod is $L = L_p/\gamma = 0.8 \text{ m}$.
- The time for this rod to pass the observer is $t = L/v = (0.8 \text{ m})/(0.6c) = 4.4 \times 10^{-9} \text{ s}$.
- Here's the spacetime diagram for this situation.



The projection OA on the x axis is L .
The projection OB on the ct axis yields t .

The x' axis is the line $ct' = 0$. The rod is at rest in S' , so the x' coordinates of the endpoints are the same for all ct' . That's why the worldlines of the endpoints are parallel to the ct' axis. (See Fig. 1-21 for the analogous situation in the S frame; the vertical lines are the worldlines of particles that never move in frame S .)

Let's work out the equation of the dotted line on the diagram, which is the worldline of the left end of the rod. The Lorentz transformation is $x' = \gamma(x - vt) = \gamma(x - \beta ct)$.

The left end of the rod is at $x' = -1$ m. We substitute that into the previous equation and also put in the values of γ and β . Then we rearrange it to solve for ct in terms of x . The result is

$$ct = \frac{5}{3}x + \frac{4}{3}.$$

If we set $ct = 0$, then and solve for x we get $x = -0.8$ m. That's the coordinate of the left end of the rod at $t = 0$. The right end is at $x = 0$ at $t = 0$, so the length of the rod is 0.8 m; that's what we got above. Similarly if we set $x = 0$, then we can solve for the time at which the left end passes the origin; it is $t = 4/(3c) = 4.4 \times 10^{-9}$ s, the same that we got above.

Problem 3: Tipler 1-42

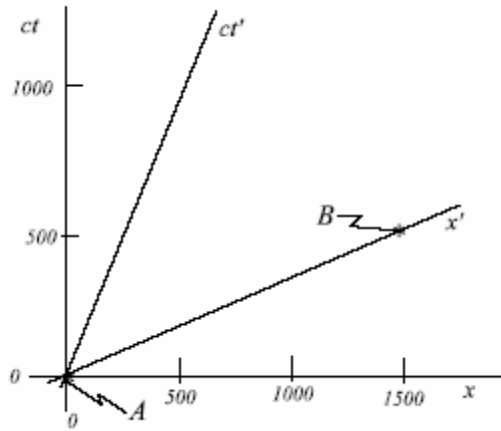
Two events: A and B with $\Delta x = x_B - x_A = 1.5 \times 10^3$ m and $\Delta t = t_B - t_A = 2 \times 10^{-6}$ s, so $c\Delta t = 600$ m.

- a) To see if there is a frame S' where the two events are simultaneous ($\Delta t' = 0$), use the Lorentz time transformation $\Delta t' = \gamma \left(\Delta t - \frac{v}{c^2} \Delta x \right)$. If this is zero, then

$$\Delta t = \frac{v}{c^2} \Delta x \text{ or}$$

$$v = \frac{c^2 \Delta t}{\Delta x} = \frac{c^2}{(\Delta x / \Delta t)} = \frac{(3 \times 10^8 \text{ m/s})}{(1.5 \times 10^3 \text{ m} / 2 \times 10^{-6} \text{ s})} c = 0.4c = 1.20 \times 10^8 \text{ m/s}.$$

- b) From the Lorentz time transformation equation written in (a), we can see that if v is larger than the value computed in (a), then $\Delta t'$ would become negative. That means the order of the events is reversed in that frame, i.e. B precedes A.
- c) Here is the spacetime diagram for this problem.



Note: B is on the x' axis, i.e. where $ct' = 0$, as is A. For any x' slope greater than 0.4, the order of B and A is reversed.

On the (x, ct) axes, the spacetime coordinates of the two events are A: (0,0) and B:(1500 m, 600 m). Then the x' axis is drawn through those points; the x' axis is also the line $ct' = 0$, so that's how on the diagram we say that the two events occur at the same time. Then we can calculate the slope of this line to be “rise/run” or $600/1500 = 0.4$, and that's the value of β . For a frame with a larger value of β , point A would still be on the line and point B would be below it, and from that you conclude that event B would occur first in that frame.

d) Calculate the spacetime interval in the original frame

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2 = \left[(3 \times 10^8 \text{ m/s})(2 \times 10^{-6}) \right]^2 - (15 \times 10^3 \text{ m})^2 = -1.89 \times 10^6 \text{ m}^2.$$

This is a spacelike interval. Notice that when the interval is spacelike, Tipler reverses the sign [look at (Eq. 1.35)], so that the square root can be taken. So here Tipler would say $\Delta s = 1370 \text{ m}$.

Since the spacetime interval is invariant (previous problem!), it has the same value in any other frame, so

$$(\Delta s)^2 = (c\Delta t')^2 - (\Delta x')^2 = -1.89 \times 10^6 \text{ m}^2.$$

If we choose the frame we found in (a), where $\Delta t' = 0$, then the spatial separation between the two events is

$$\begin{aligned} -(\Delta x')^2 &= -1.89 \times 10^6 \text{ m}^2, \\ \Delta x' &= 1370 \text{ m}. \end{aligned}$$

Because the two events are simultaneous in this frame, this distance is called the proper distance between the events, L_p .

Problem 4: Tipler 1-57

$$v = 0.6c \quad \gamma = \frac{1}{\sqrt{1 - (0.6)^2}} = 1.25$$

(a) The clock in S reads $\gamma \times 60 \text{ min} = 75 \text{ min}$ when the S' clock reads 60 min and the first signal from S' is sent. At that time, the S' observer is at $v \times 75 \text{ min} = 0.6c \times 75 \text{ min} = 45c \cdot \text{min}$. The signal travels for 45 min to reach the S observer and arrives at $75 \text{ min} + 45 \text{ min} = 120 \text{ min}$ on the S clock.

(b) The observer in S sends his first signal at 60 min and its subsequent wavefront is found at $x = c(t - 60 \text{ min})$. The S' observer is at $x = vt = 0.6ct$ and receives the wavefront when these x positions coincide, i.e., when

$$\begin{aligned} c(t - 60 \text{ min}) &= 0.6ct \\ 0.4ct &= 60c \cdot \text{min} \\ t &= (60c \cdot \text{min}) / 0.4c = 150 \text{ min} \\ x &= 0.6c(150 \text{ min}) = 90c \cdot \text{min} \end{aligned}$$

The confirmation signal sent by the S' observer is sent at that time and place, taking 90 min to reach the observer in S. It arrives at $150 \text{ min} + 90 \text{ min} = 240 \text{ min}$.

(c) Observer in S:

Sends first signal	60 min
Receives first signal	120 min
Receives confirmation	240 min

The S' observer makes identical observations.