

## 2006-09-22 Problem Solutions

### **Problem 1: Tipler 2-23**

Use conservation of both energy and momentum to solve this. The original kaon is at rest, so has zero momentum. The total momentum of the two pions must also be zero, and therefore they must move off in opposite directions with momenta of the same magnitude. Since the two particles after the decay are the same kind, they have the same rest mass energy. Since the two pions have the same (magnitude of) momenta and the same rest mass energy, they must have the same energy [Eq. (2-36)]. Each of their energies must be one-half of the rest mass energy of the original stationary kaon:  $E_\pi = 497.7 \text{ MeV}/2 = 248.9 \text{ MeV}$ . Now we look up the rest mass energy of a charged pion (pi meson) in Table 2-1, page 87:  $m_\pi c^2 = 139.6 \text{ MeV}$ . Therefore the kinetic energy of each of these pions is  $E_k = (248.9 - 139.6) \text{ MeV} = 109.3 \text{ MeV}$ .

### **Problem 2: Tipler 2-25**

The book discusses annihilation of an electron-positron pair to form two photons on p. 95 at about the middle of the page. The two photons are emitted in opposite directions and each has energy of 0.511 MeV. The total energy is  $2(0.511 \text{ MeV}) = 1.022 \text{ MeV}$ , and the total momentum is 0, since the photons move in opposite directions. Therefore the invariant energy is

$$E_{total}^2 - (p_{total}c)^2 = [2(0.511 \text{ MeV})]^2. \quad (1.1)$$

This is the same as the square of the total rest energy of an electron-positron pair, since for each one  $mc^2 = 0.511 \text{ MeV}$ .

### **Problem 3: Tipler 2-29**

Answer the second question first, by using the relativistic energy-momentum relation to find the rest mass energy.

$$\begin{aligned} E^2 &= (mc^2)^2 + (cp)^2 \\ \Rightarrow (mc^2)^2 &= E^2 - (cp)^2 = (1746 \text{ MeV})^2 - \left(500 \frac{\text{Mev}}{c} c\right)^2 = 2798516 \text{ MeV}^2 \quad (1.2) \\ \Rightarrow mc^2 &= 1673 \text{ MeV} \quad \Rightarrow m = 1673 \frac{\text{MeV}}{c^2}. \end{aligned}$$

Now get the speed from

$$\frac{u}{c} = \frac{cp}{E} = \frac{500 \text{ Mev}}{1746 \text{ Mev}} = 0.286. \quad (1.3)$$

#### **Problem 4: Tipler 2-39**

First, we get the relativistic factor  $1/[1 - (u/c)^2]^{1/2}$ , from the energy formula

$$E = \frac{mc^2}{\sqrt{1 - \left(\frac{u}{c}\right)^2}}, \quad (50.00 \text{ Gev}) = \frac{0.511 \text{ Mev}}{\sqrt{1 - \left(\frac{u}{c}\right)^2}}, \quad \frac{1}{\sqrt{1 - \left(\frac{u}{c}\right)^2}} = \frac{50000}{0.511} = 9.78 \times 10^4. \quad (4)$$

- a) The question asks for the proper length of the electron/positron bunches. Their length in the lab frame is  $L = 0.01 \text{ m}$ . The length of the bunch in the lab frame  $S$  is related to the proper length in frame  $S'$  of the bunch by the length contraction formula

$$L = L_p \sqrt{1 - \left(\frac{u}{c}\right)^2}, \quad \text{or} \quad L_p = \frac{1}{\sqrt{1 - \left(\frac{u}{c}\right)^2}} L, \quad (5)$$

$$L_p = (9.78 \times 10^4)(0.01 \text{ m}) = 978 \text{ m}.$$

The transverse width has the same value as in the rest frame of one of the bunches,  $10 \text{ } \mu\text{m}$ .

- b) [This is sort of opposite to the pole-barn problem. The proper length of the “pole” (bunch) is shorter than the proper length of the “barn” (accelerator).] We want the contracted length of the accelerator  $L_a$  to be the size of the proper length of the bunch just computed. The speed of the accelerator relative to the particle bunches is the same as the speed of the bunches relative to the accelerator. Therefore

$$L_a = \sqrt{1 - \left(\frac{u}{c}\right)^2} L_{a,p}, \quad \text{or} \quad 978 \text{ m} = \frac{1}{9.78 \times 10^4} L_{a,p}, \quad \text{so} \quad (6)$$

$$L_{a,p} = (9.78 \times 10^4)(978 \text{ m}) = 9.56 \times 10^7 \text{ m}.$$

( $L_{a,p}$  is the proper length of the accelerator.) This length is huge, about 2.5 times the circumference of Earth.

- c) The positron bundle is  $10^{-2}$  m long in the lab frame, so in the electron's frame it is contracted to  $L = (10^{-2} \text{ m})/9.78 \times 10^4 = 1.02 \times 10^{-7} \text{ m}$ .
- d) Use the momentum and energy transformation formulas to answer this. The electrons and positrons go in opposite direction around the accelerator (because they have opposite charge). So it looks like this:



Suppose the electron is on the left and the positron on the right. Frame  $S'$  is attached to the positron, so its velocity relative to the lab frame  $S$  is  $v = -u$ , where  $u$  is the speed of the either particle. Then

$$\gamma = \left[ 1 - \left( \frac{v}{c} \right)^2 \right]^{-1/2} = \left[ 1 - \left( \frac{-u}{c} \right)^2 \right]^{-1/2} = 9.78 \times 10^4 \quad (7)$$

was found at the beginning of the problem. We need to turn this around to find  $v/c$ . Since  $\gamma$  is so large, we use the approximation from the binomial expansion (this formula was worked out on an earlier homework set)

$$\frac{u}{c} = 1 - \frac{1}{2\gamma^2} = 1 - 5.227 \times 10^{-11} \approx 1 = -\frac{v}{c}. \quad (8)$$

I.e. for almost all practical purposes we can take the speed of  $S'$  relative to  $S$  to be  $c$ . But we can't use this value in formulas involving  $\gamma = [1 - u^2/c^2]^{-1/2}$ .

Next we need the momentum in the lab frame. Since the total energy  $E = 50 \text{ GeV}$  is so much larger than the rest mass energy  $0.511 \text{ MeV}$ , we use the approximation  $E = pc$  to get

$$p = \frac{E}{c} = 50 \frac{\text{GeV}}{c}. \quad (9)$$

Now we can use the momentum and energy transformation formulas

$$\begin{aligned} cp' &= \gamma \left( cp - \frac{v}{c} E \right) = (9.78 \times 10^4) [50 \text{ GeV} - (-1)(50 \text{ GeV})] \\ &= (9.78 \times 10^4)(100 \text{ GeV}) = 9.78 \times 10^6 \text{ GeV}, \\ \text{so } p' &= 9.78 \times 10^6 \frac{\text{GeV}}{c}. \end{aligned} \quad (10)$$

$$\begin{aligned}
 E' &= \gamma(E - vp) = \gamma \left( E - \frac{v}{c} cp \right) = (9.78 \times 10^4) [50 \text{ Gev} - (-1) 50 \text{ Gev}] \\
 &= 9.78 \times 10^6 \text{ Gev}.
 \end{aligned}
 \tag{11}$$

To two-decimal-place accuracy  $E$  and  $pc$  are equal. That's because  $E$  is much greater than the rest-mass energy  $mc^2$ .