

2006-09-27 Problem Solutions

Tipler, problem 2-42

(a) $p = 300BR(q/e)$ (Equation 2-38)

$$p = 300(1.5\text{ T})(6.37 \times 10^6)(1) = 2.87 \times 10^9 \text{ MeV}/c$$

For $E \gg mc^2$, $E = E_k$ and $E = pc$ (Equation 2-32) $\therefore E_k = pc = 2.87 \times 10^9 \text{ MeV}$

(b) For $E = pc$, $u=c$ and

$$T = 2\pi R/c = 2\pi(6.37 \times 10^6 \text{ m})/c = 0.133 \text{ s}$$

Tipler, problem 3-1

First, compare this velocity with c , to see if relativistic effects are important.

$$\frac{v}{c} = \frac{2.5 \times 10^6}{3 \times 10^8} = 0.833 \times 10^{-2} < 1\% \quad (1)$$

This value indicates we can use non-relativistic formulas for momentum and kinetic energy. From formulas for centripetal force and magnetic force

$$\frac{mv^2}{r} = qvB \quad \Rightarrow \quad r = \frac{mv}{qB}. \quad (2)$$

[Compare with the relativistic version of this formula in Eq. (2-37).] All of the particles in this problem have the same *magnitude* of charge, $|e| = 1.6 \times 10^{-19} \text{ C}$, they all have the same velocity $v = 2.5 \times 10^6 \text{ m/s}$, and they all move in the same magnetic field, $B = 0.40 \text{ T}$. We have to find the different masses of each one, substitute them into Eq. (2) and calculate r . Let's put all the data in a table. [The masses can all be found in the Appendices of the book.]

PARTICLE	MASS	RADIUS OF PATH
Proton	$1.67 \times 10^{-27} \text{ kg}$	$6.52 \times 10^{-2} \text{ m}$
Electron	$9.11 \times 10^{-31} \text{ kg}$	$3.56 \times 10^{-5} \text{ m}$

Deuteron	$3.34 \times 10^{-27} \text{ kg}$	$1.30 \times 10^{-1} \text{ m}$
He^+	$6.64 \times 10^{-27} \text{ kg}$	$2.60 \times 10^{-1} \text{ m}$
H_2^+	$3.37 \times 10^{-27} \text{ kg}$	$1.32 \times 10^{-1} \text{ m}$

Problem 3: Tipler 3-5

Here is the solution given in the answer book for our textbook.

Compare the kinetic energy to the rest energy for this electron: $(45 \text{ keV})/(0.511 \text{ MeV}) = 8.8 \times 10^{-2} < 10\%$. This is about the upper limit of validity of non-relativistic formulas.

- a) First, find the velocity. (This formula is non-relativistic; the factors of c are put in solely to be able to write the electron mass in energy units, which we all know is 0.511 MeV .)

$$\begin{aligned}
 K = \frac{1}{2} mu^2 &= \frac{1}{2} mc^2 \left(\frac{u}{c} \right)^2 \quad \Rightarrow \quad 45 \times 10^3 \text{ eV} = \frac{1}{2} (0.511 \times 10^6 \text{ eV}) \left(\frac{u}{c} \right)^2 \\
 \Rightarrow \left(\frac{u}{c} \right)^2 &= \frac{2(45 \times 10^3 \text{ eV})}{(0.511 \times 10^6 \text{ eV})} = 1.76 \times 10^{-1} \quad \Rightarrow \quad \frac{u}{c} = 4.20 \times 10^{-1} \\
 \Rightarrow u &= 1.26 \times 10^8 \frac{\text{m}}{\text{s}}.
 \end{aligned} \tag{3}$$

Find the radius from formula in preceding problem.

$$r = \frac{mu}{qB} = \frac{(9.11 \times 10^{-31} \text{ kg})(1.26 \times 10^8 \text{ m/s})}{(1.6 \times 10^{-19} \text{ C})(0.325 \text{ T})} = 2.21 \times 10^{-3} \text{ m}. \tag{4}$$

- b) The period is the time to go around the circle once,

$$T = \frac{2\pi r}{u} = \frac{2(3.141)(2.21 \times 10^{-3} \text{ m})}{1.26 \times 10^8 \text{ m/s}} = 1.10 \times 10^{-10} \text{ s} = 0.110 \text{ ns}. \tag{5}$$

The frequency is

$$f = \frac{1}{T} = 9.07 \times 10^9 \text{ Hz} = 9.07 \text{ GHz}. \tag{6}$$

However, this solution is highly suspect, because the speed of this electron is 45% of the speed of light. Let's do it relativistically.

The kinetic energy is $E_k = 45 \text{ keV} = 0.045 \text{ MeV}$, and the rest energy of an electron is 0.511 MeV , so the total energy is $E = 0.556 \text{ MeV}$. Therefore

$$E = \gamma mc^2 \text{ with } \gamma = \frac{1}{\sqrt{1 - \left(\frac{u}{c}\right)^2}} \text{ gives } \gamma = \frac{0.556}{0.511} = 1.088. \quad (7)$$

Then the speed is

$$\frac{u}{c} = \sqrt{1 - \frac{1}{\gamma^2}} = 0.394 \quad \text{so} \quad u = 1.18 \times 10^8 \text{ m/s}. \quad (8)$$

This is quite a bit smaller than the value found above. The momentum is

$$cp = \gamma mcu \quad \text{so} \quad cp = \gamma mc^2 \left(\frac{u}{c}\right) = (0.556 \text{ MeV})(0.394) = 0.219 \text{ MeV}, \quad (9)$$

$$\Rightarrow \quad p = 0.219 \frac{\text{MeV}}{c}.$$

We get the radius from Eq. (2-38), [r in m, p in MeV/c, q in number of electron charges]

$$r = \frac{p}{300B \frac{q}{e}} = \frac{0.219}{(300)(0.325)(1)} = 0.00225 \text{ m}. \quad (10)$$

The period is

$$T = \frac{2\pi r}{u} = \frac{2(3.141)(2.225 \times 10^{-3} \text{ m})}{1.18 \times 10^8 \text{ m/s}} = 1.18 \times 10^{-10} \text{ s} = 0.118 \text{ ns}, \quad (11)$$

and the frequency is $f = 1/T = 8.44 \text{ GHz}$. So there is some difference.

Problem 4: Tipler 3-11

a) The terminal speed when falling is

$$v_f = \frac{mg}{b}, \quad (12)$$

where the coefficient in the friction force is from Stokes' Law

$$b = 6\pi\eta a. \quad (13)$$

Combine Eqs. (12) and (13) and get

$$v_f = \frac{\left(\frac{4}{3}\pi a^3 \rho\right) g}{6\pi\eta a} = \left(\frac{2}{9} \frac{\pi \rho g}{\eta}\right) a^2 \quad \Rightarrow \quad a^2 = \frac{9}{2} \frac{\eta v_f}{\rho g}. \quad (14)$$

In the 1st step, the mass was written as the volume of a sphere times the mass density of oil. The specific gravity gives the ratio of the density of the oil to the density of water. Put in the numbers and get

$$a = \sqrt{\frac{9}{2} \frac{(1.8 \times 10^{-5} \text{ N} \cdot \text{s/m}^2)(5 \times 10^{-3} \text{ m} / 20 \text{ s})}{(0.75 \times 1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)}} = 1.66 \times 10^{-6} \text{ m} = 1.66 \text{ micron}. \quad (15)$$

Now that we know a , the mass is

$$m = \frac{4}{3} \pi a^3 \rho = 1.44 \times 10^{-14} \text{ kg}. \quad (16)$$

b) The ratio of these two forces is

$$\frac{F_E}{F_G} = \frac{neE}{mg} = \frac{(2)(1.6 \times 10^{-19} \text{ C})(2.5 \times 10^5 \text{ N/C})}{(1.44 \times 10^{-14} \text{ kg})(9.8 \text{ m/s}^2)} = 0.57. \quad (17)$$

This is an interesting number (!). If you compare the electrical and gravitational forces in other situations, e.g. between the electron and proton in a hydrogen atom, you get a huge ratio, approximately 10^{40} . Part of Millikan's cleverness in this experiment was to get a small amount of charge (a few extra electrons) onto a mass of "macroscopic" size.