

2006-09-29 Problem Solutions

Problem 1: Tipler 2-49

Energy conservation: $E_\mu + E_\nu = M_\pi c^2$, since the pion decayed at rest.

Momentum conservation: $p_\mu + p_\nu = 0$, i.e. the muon and neutrino have equal and opposite momenta, again because the pion had zero momentum before decay.

In the energy conservation equation, use the formula for energy in terms of momentum and rest mass energy. Since in this part we're assuming that the neutrino has no mass, its energy is just $p_\nu c$.

$$\sqrt{(M_\mu c^2)^2 + (p_\mu c)^2} + (p_\nu c) = M_\pi c^2$$

The terms $(p_\mu c)^2$ and $(p_\nu c)^2$ have the same magnitude, because of the momentum conservation equation; call it $(pc)^2$ for the moment. Rearrange the previous equation.

$$\sqrt{(M_\mu c^2)^2 + (pc)^2} = M_\pi c^2 - (p_\nu c)$$

$$(M_\mu c^2)^2 + (pc)^2 = (M_\pi c^2)^2 - 2(M_\pi c^2)(pc) + (pc)^2$$

$$pc = \frac{(M_\pi c^2)^2 - (M_\mu c^2)^2}{2(M_\pi c^2)} = \frac{(139.56766 \text{ MeV})^2 - (105.65839 \text{ MeV})^2}{2(139.56766 \text{ MeV})} = 29.7899 \text{ MeV}$$

$$p = 29.7899 \frac{\text{MeV}}{c}$$

This is the magnitude of both the muon and neutrino momentum; they're directed in opposite directions. The energies are

$$E_\nu = pc = 29.7899 \text{ MeV}$$

$$E_\mu = \sqrt{(105.65839 \text{ MeV})^2 + (29.7899 \text{ MeV})^2} = 109.7777 \text{ MeV}$$

b) If the neutrino has non-zero rest mass, the energy conservation equation is

$$M_\pi c^2 = E_\mu + E_\nu = \sqrt{(M_\mu c^2)^2 + (p_\mu c)^2} + \sqrt{(M_\nu c^2)^2 + (p_\nu c)^2}$$

The two momenta terms have the same magnitude, as in (a).

$$\begin{aligned}
M_{\pi}c^2 &= \sqrt{(M_{\mu}c^2)^2 + (pc)^2} + \sqrt{(M_{\nu}c^2)^2 + (pc)^2} \\
M_{\pi}c^2 - \sqrt{(M_{\nu}c^2)^2 + (pc)^2} &= \sqrt{(M_{\mu}c^2)^2 + (pc)^2} \\
(M_{\pi}c^2)^2 - 2(M_{\pi}c^2)^2 \sqrt{(M_{\nu}c^2)^2 + (pc)^2} + (M_{\nu}c^2)^2 + (pc)^2 &= (M_{\mu}c^2)^2 + (pc)^2
\end{aligned}$$

The $(pc)^2$ terms cancel. Collect all the rest mass energy terms on one side.

$$\begin{aligned}
\sqrt{(M_{\nu}c^2)^2 + (pc)^2} &= \frac{(M_{\pi}c^2)^2 + (M_{\nu}c^2)^2 - (M_{\mu}c^2)^2}{2(M_{\pi}c^2)^2} \\
\sqrt{(0.190 \text{ MeV})^2 + (pc)^2} &= \frac{(139.56755 \text{ MeV})^2 - (105.65839 \text{ MeV})^2 + (0.190 \text{ MeV})^2}{2(139.56755 \text{ MeV})} \\
&= 29.7900 \text{ MeV} \\
(0.190 \text{ MeV})^2 + (pc)^2 &= (29.7900 \text{ MeV})^2 = 887.410 \text{ MeV}^2 \\
pc &= 29.7894 \text{ MeV} \\
p &= 29.7894 \frac{\text{MeV}}{c}
\end{aligned}$$

This small value of the neutrino mass makes a difference in the 4th decimal place in the parameters of this decay process. This problem illustrates why it was unclear for a long time whether or not neutrinos have mass.

The energy values are essentially the same as in (a).

See the end of this document for another solution of this problem.

Problem 2: Tipler 3-2

(a) Using ^{198}Au mass number to compute an approximate value for B that will yield R of one meter, Equation 3-2 gives

$$B = \frac{mu}{qR} = m \left[\frac{(198u)(1.66 \times 10^{-27} \text{ kg/u})(1.5 \times 10^5 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(1.0 \text{ m})} \right] = 0.31 \text{ T}$$

(b) Using Equation 3-2, $R = \frac{mu}{qB} \rightarrow \Delta R = \Delta m \left(\frac{u}{qB} \right)$

$$\Delta R = (m_{\text{Hg}} - m_{\text{Au}}) \left(\frac{u}{qB} \right) = (m_{\text{Hg}} - m_{\text{Au}}) \left[\frac{(1.66 \times 10^{-27} \text{ kg/u})(1.5 \times 10^5 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.31 \text{ T})} \right]$$

$$\Delta R = (197.966743u - 196.966543u)(5.02 \times 10^{-3} \text{ m/u}) = 0.005 \text{ m} = 5 \text{ mm}$$

(c) For doubly ionized atoms $q = 2 \times 1.60 \times 10^{-19} \text{ C}$, so $B = 0.31 \text{ T}/2 = 0.16 \text{ T}$ and ΔR is unchanged because

$$qB = (2)(1.6 \times 10^{-19} \text{ C})(0.31 \text{ T}/2) = (1.60 \times 10^{-19} \text{ C})(0.31 \text{ T}) \text{ as before.}$$

Problem 3: Tipler 3-3

$$B = \frac{\mathcal{E}}{u}, \frac{u}{c} = \frac{pc}{E}, \text{ and } pc = \sqrt{E^2 - (mc^2)^2}$$

$$pc = \sqrt{(0.561 \text{ MeV})^2 - (0.511 \text{ MeV})^2} = 0.2315 \text{ MeV}$$

$$\frac{u}{c} = \frac{0.2315 \text{ MeV}}{0.561 \text{ MeV}} = 0.41$$

$$\therefore B = \frac{2.0 \times 10^5 \text{ V/m}}{0.41c} = 1.63 \times 10^{-3} \text{ T} = 16.3 \text{ G}$$

Problem 4: Tipler 3-9

For the rise time to equal the field-free fall time, the net upward force must equal the weight.

$$q\mathcal{E} - mg = mg \therefore \mathcal{E} = 2mg/q.$$

Here's the solution to Problem 1: Tipler 2-49 from the authors' solution manual. They do part (a) by calculating γ . In part (b), I don't know where the 250 keV comes from. My book says the rest mass energy of the neutrino is 190 keV.

(a) If ν mass is 0:

$$E_{\mu}^2 = (p_{\mu}c)^2 + (m_{\mu}c^2)^2 \text{ and } E_{\nu}^2 = (p_{\nu}c)^2 + 0$$

$$E_{k\mu} + E_{\nu} = 139.56755 \text{ MeV} - 105.65839 \text{ MeV}$$

$$m_{\mu}c^2(\gamma - 1) + E_{\nu} = 33.90916 \text{ MeV}$$

$$p_{\mu}c = (E_{\mu}^2 - (m_{\mu}c^2)^2)^{1/2} = 33.90916 - m_{\mu}c^2(\gamma - 1)$$

Squaring, we have

$$(m_{\mu}c^2)^2(\gamma^2 - 1) = (33.90916)^2 - 2(33.90916)(m_{\mu}c^2)(\gamma - 1) + (m_{\mu}c^2)^2(\gamma - 1)^2$$

Collecting terms, then solving for $(\gamma - 1)$,

$$\gamma - 1 = \frac{(33.90916)^2}{2(m_{\mu}c^2)^2 + 2(33.90916)m_{\mu}c^2} \quad \text{Substituting } m_{\mu}c^2 = 105.65839 \text{ MeV},$$

$$\gamma - 1 = 0.0390 \rightarrow \gamma = 1.0390 \text{ so,}$$

$$E_{k\mu} = 4.12 \text{ MeV} \text{ and } p_{\mu} = \frac{1}{c}[(109.78)^2 - (105.66)^2]^{1/2} = 29.8 \text{ MeV}/c$$

$$E_{\nu} = 29.8 \text{ MeV} \text{ and } p_{\nu} = 29.8 \text{ MeV}/c$$

(b) If ν_{μ} mass = 250 keV, then $E_{\nu}^2 = (p_{\nu}c)^2$ and

$$E_{k\mu} + E_{k\nu} = 139.56755 \text{ MeV} - 105.65839 \text{ MeV} - 0.250 \text{ MeV} = 33.67916 \text{ MeV}$$

Solving as in (a) yields

$$E_{\mu} = 109.78 \text{ MeV}, p_{\mu} = 29.8 \text{ MeV}/c, E_{\nu} = 29.8 \text{ MeV}, p_{\nu} = 29.8 \text{ MeV}/c$$