

2006-10-04 Problem Solutions

Useful fact for these problems: Planck's constant in eV·s: $h = 4.136 \times 10^{-15} \text{ eV} \cdot \text{s}$.

Tipler 3-26

Basic photoelectric effect equation: $K_{\max} = hf - \phi$, $K_{\max}$ is maximum kinetic energy of ejected electrons.

- a) $\phi = 1.9 \text{ eV}$ Threshold frequency occurs at $K_{\max} = 0$.

$$f_{\text{thresh}} = \frac{\phi}{h} = \frac{1.9 \text{ eV}}{4.136 \times 10^{-15} \text{ eV} \cdot \text{s}} = 4.59 \times 10^{14} \text{ Hz}$$

$$\lambda = \frac{c}{f} = \frac{2.998 \times 10^8 \text{ m/s}}{4.59 \times 10^{14} \text{ s}^{-1}} = 6.53 \times 10^{-7} \text{ m} = 0.653 \text{ micron}$$

- b) The stopping potential is related to the maximum KE of the ejected electrons by $eV_s = K_{\max}$, so $eV_s = hf - \phi$ or $V_s = \frac{hf - \phi}{e}$.

$$\lambda = 300 \text{ nm} = 300 \times 10^{-9} \text{ m} \qquad f = \frac{2.998 \times 10^8 \text{ m/s}}{300 \times 10^{-9} \text{ m}}$$

$$V_s = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(9.993 \times 10^{14} \text{ Hz}) - 1.9 \text{ eV}}{e} = \frac{2.233 \text{ eV}}{e} = 2.233 \text{ Volts}.$$

- c) $\lambda = 400 \times 10^{-9} \text{ m} \qquad f = \frac{2.998 \times 10^8 \text{ m/s}}{400 \times 10^{-9} \text{ m}} = 7.495 \times 10^{14} \text{ Hz}$

$$V_s = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(7.495 \times 10^{14} \text{ Hz}) - 1.9 \text{ eV}}{e} = \frac{1.200 \text{ eV}}{e} = 1.200 \text{ Volts}$$

Tipler 3-29

Photon energy $E = hf = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda}$. The useful fact $hc = 1240 \text{ eV} \cdot \text{nm}$ is given in Example 3-6.

- a) $\lambda = 0.1 \text{ nm} \qquad E = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{0.1 \text{ nm}} = 12400 \text{ eV}$

- b) $\lambda = 1 \text{ fm} = 10^{-15} \text{ m} = 10^{-6} \text{ nm} \qquad E = \frac{1240 \text{ eV} \cdot \text{nm}}{10^{-6} \text{ nm}} = 1.240 \times 10^9 \text{ eV} = 1.240 \text{ GeV}$

$$\text{c) } f = 90.7 \text{ MHz} \quad \lambda = \frac{2.998 \times 10^8 \text{ m/s}}{90.7 \times 10^6 \text{ Hz}} = 3.305 \text{ m}$$

$$E = \frac{1240 \text{ eV} \cdot \text{nm}}{3.305 \times 10^9 \text{ nm}} = 3.750 \times 10^{-7} \text{ eV}$$

Tipler 3-42

Use the photoelectric equation, $eV_s = hf - \phi = \frac{hc}{\lambda} - \phi$.

$$\text{We have two sets of numbers for this equation:}$$

$$e(0.52 \text{ V}) = \frac{hc}{450 \text{ nm}} - \phi$$

$$e(1.90 \text{ V}) = \frac{hc}{300 \text{ nm}} - \phi$$

Subtract the first from the second equation to make ϕ disappear:

$$e(1.38 \text{ V}) = hc \left(-\frac{1}{450 \text{ nm}} + \frac{1}{300 \text{ nm}} \right) = hc (1.111 \times 10^{-3} \text{ nm}^{-1})$$

$$h = \frac{e(1.38 \text{ V})}{1.111 \times 10^{-3} \text{ nm}^{-1}} = \frac{(1.602 \times 10^{-19} \text{ C}) \left(1.38 \frac{\text{J}}{\text{C}} \right)}{(1.111 \times 10^{-3} \text{ nm}^{-1}) \left(3 \times 10^8 \text{ m/s} \cdot 10^9 \frac{\text{nm}}{\text{m}} \right)} = 6.663 \times 10^{-34} \text{ J} \cdot \text{s}$$

Now go back to either one of the starting equations to get ϕ .

$$\phi = \frac{hc}{450 \text{ nm}} - e(0.52 \text{ V}) \Rightarrow \phi = \frac{1240 \text{ eV} \cdot \text{nm}}{450 \text{ nm}} - 0.52 \text{ eV} = 2.24 \text{ eV}$$

Tipler 3-46

Think of this as a collision, in which a photon collides with a free electron and is absorbed. Consider this problem in the “center of mass” frame. Before the collision the photon (subscript γ , for gamma rays) and the free electron (subscript e) have momenta of equal magnitude and opposite direction,

$$p_\gamma + p_e = 0. \quad (1)$$

The total energy before the collision in this frame is

$$E_{\text{total}} = E_\gamma + \sqrt{(m_e c^2)^2 + (p_e c)^2} \quad (2)$$

After the collision there is only an electron, and it is at rest, since its total momentum must be zero. Therefore its energy is just its rest energy, so conservation of energy gives

$$E_\gamma + \sqrt{(m_e c^2)^2 + (p_e c)^2} = m_e c^2. \quad (3)$$

Rewrite Eq. (3):

$$E_\gamma - (m_e c^2) = -\sqrt{(m_e c^2)^2 + (p_e c)^2}, \quad (4)$$

square both sides

$$E_\gamma^2 - 2(m_e c^2)E_\gamma + \underbrace{(m_e c^2)^2} = \underbrace{(m_e c^2)^2} + (p_e c)^2. \quad (5)$$

The underlined terms cancel. Also, because of momentum conservation $p_e = -p_\gamma$, so Eq. (5) becomes

$$E_\gamma^2 - 2(m_e c^2)E_\gamma = (p_\gamma c)^2. \quad (6)$$

But for photons $E_\gamma = p_\gamma c$, so the first term on the left cancels the term on the right. This leaves

$$2(m_e c^2)E_\gamma = 0. \quad (7)$$

The solution to this is $E_\gamma = 0$. But if the photon energy equals zero, there isn't any photon. Therefore there is no non-trivial solution to this problem that satisfies the conservation laws. Absorption of a photon by a free electron is impossible.