

2006-10-06 Problem Solutions

A very useful number for doing these problems is $hc = 1240 \text{ eV} \cdot \text{nm}$.

Problem 1: Tipler 3-30

Using Equation 3-36,

$$(1) \quad 0.95 = \frac{h}{e} \left(\frac{c}{435.8 \times 10^{-9} \text{ m}} \right) - \frac{\phi}{e}$$

$$(2) \quad 0.38 = \frac{h}{e} \left(\frac{c}{546.1 \times 10^{-9} \text{ m}} \right) - \frac{\phi}{e}$$

Subtracting (2) from (1),
$$0.57 = \frac{hc}{e 10^{-9}} \left(\frac{1}{435.8} - \frac{1}{546.1} \right)$$

Solving for h yields: $h = 6.56 \times 10^{-34} \text{ J} \cdot \text{s}$. Substituting h into either (1) or (2) and solving for ϕ/e yields: $\phi/e = 1.87 \text{ eV}$. Threshold frequency is given by $hf/e = \phi/e$ or

$$f = \left(\frac{\phi}{e} \right) \left(\frac{e}{h} \right) = \frac{(1.87 \text{ eV})(1.60 \times 10^{-19} \text{ C})}{6.56 \times 10^{-34} \text{ J} \cdot \text{s}} = 4.57 \times 10^{14} \text{ Hz}$$

Problem 2: Tipler 3-36

The energy of this photon is $E = 0.511 \text{ MeV}$, so its momentum is $p = 0.511 \frac{\text{MeV}}{c}$.

The wavelength is $p = \frac{h}{\lambda}$ so $\lambda = \frac{h}{p}$, and its initial wavelength is

$$\lambda_i = \frac{h}{0.511 \frac{\text{MeV}}{c}} = \frac{hc}{0.511 \text{ MeV}} = \frac{1240 \text{ eV} \cdot \text{nm}}{0.511 \text{ MeV}} = 2.427 \times 10^{-3} \text{ nm} = 2.427 \text{ pm}.$$

The final wavelength is given by the Compton effect formula $\lambda_f - \lambda_i = \frac{h}{m_e c} (1 - \cos \theta)$.

For an electron the Compton wavelength is (p. 147) $\frac{h}{m_e c} = 0.00243 \text{ nm}$. Therefore the

final wavelength, for scattering at 110° is

$\lambda_f = 2.427 \times 10^{-3} \text{ nm} + (0.00243 \text{ nm})(1 - \cos 110^\circ) = 5.688 \times 10^{-3} \text{ nm}$. This gives the final momentum of the photon to be

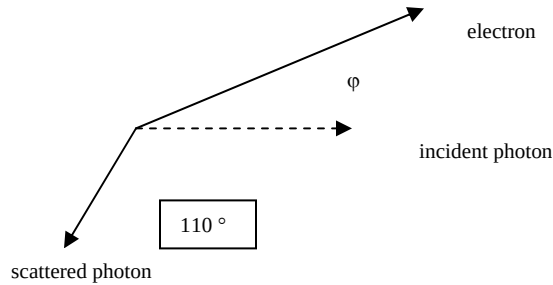
$$p_f = \frac{h}{\lambda_f} \quad \text{or} \quad cp_f = \frac{hc}{\lambda_f} = \frac{1240 \text{ eV} \cdot \text{nm}}{5.688 \times 10^{-3} \text{ nm}} = 0.218 \text{ MeV}. \quad \text{Therefore the momentum of}$$

the final (scattered) photon is $p_f = 0.218 \frac{\text{MeV}}{c}$. The energy of the scattered photon is

$$E_f = p_f c = 0.218 \text{ MeV}.$$

The energy of the *initial* photon was given as $E_i = 0.511 \text{ MeV}$, and the initial electron at rest had energy 0.511 MeV ; the total energy of the initial state is 1.022 MeV . In the final state the photon had energy 0.218 MeV , so by energy conservation the recoiling electron had the rest of it, which $E_e = (1.022 - 0.218) \text{ MeV} = 0.804 \text{ MeV}$ (this is total energy).

To find the direction of the recoiling electron, first find the magnitude of its momentum from the energy momentum relation $E^2 = (mc^2)^2 + (pc)^2$. Turn it around and apply it to the final state of the electron, $p_{f,e}c = [E_f^2 - (mc^2)^2]^{1/2} = [(0.804)^2 - (0.511)^2]^{1/2} \text{ MeV} = 0.621 \text{ MeV}$. So the momentum of the recoiling electron is $p_{f,e} = 0.621 \text{ MeV}/c$. To get the angle we have to look at the picture of the scattered photon and the recoiling electron. The problem says that the scattered photon comes out at 110° , so it looks something like this.



Write the momentum conservation law for the y-component of momentum.

$$0 = p_{f,e} \sin \varphi - p_f \sin 110^\circ = 0.621 \sin \varphi - 0.218 \sin 110^\circ.$$

Solve this for the angle of the recoiling electron

$$\varphi = \arcsin \left[\frac{0.218 \sin(110^\circ)}{0.621} \right] = \arcsin(0.330) = 19.3^\circ.$$

Problem 3: Tipler 3-38

a) The energy of the incident photons is

$$E_i = hf_i = \frac{hc}{\lambda_i} = \frac{1240 \text{ eV} \cdot \text{nm}}{0.0711 \text{ nm}} = 1.747 \times 10^4 \text{ eV}.$$

b) The wavelength of the incident photons is given to be $\lambda_i = 0.0711 \text{ nm}$. The change in wavelength is given by the Compton scattering equation

$$\Delta\lambda = \frac{h}{mc}(1 - \cos \theta) = (0.00243 \text{ nm})(1 - \cos 180^\circ) = 0.00486 \text{ nm}, \text{ and so the final}$$

wavelength is $\lambda_f = \lambda_i + \Delta\lambda = (0.0711 + 0.00486) \text{ nm} = 0.07596 \text{ nm}$.

c) The energy is $E_f = \frac{hc}{\lambda_f} = \frac{1240 \text{ eV} \cdot \text{nm}}{0.07596 \text{ nm}} = 1.632 \times 10^4 \text{ eV}.$

- d) By energy conservation, the difference of energy between the initial and final photons is the electron energy. $E_e = E_i - E_f = 1.128 \times 10^3 \text{ eV}$.

Problem 4: Tipler 3-55

- (a) The nonrelativistic expression for the kinetic energy of the recoiling nucleus is

$$E_k = \frac{p^2}{2m} = \frac{(15 \text{ MeV}/c)^2}{2 \times 12u} \left(\frac{1u}{931.5 \text{ MeV}/c^2} \right) = 1.10 \times 10^4 \text{ eV}$$

$$\text{Internal energy } U = 15 \text{ MeV} - 0.0101 \text{ MeV} = 14.9899 \text{ MeV}$$

- (b) the nucleus must recoil with momentum equal to that of the emitted photon, about 14.98 MeV/c.

$$E_k = \frac{p^2}{2m} = \frac{(14.98 \text{ MeV}/c)^2}{2 \times 12u} \left(\frac{1u}{931.5 \text{ MeV}/c^2} \right) = 1.00 \times 10^{-2} \text{ eV}$$

$$E_\gamma = U - E_k = 14.9899 \text{ MeV} - 0.0100 \text{ MeV} = 14.9799 \text{ MeV}$$