

## 2006-10-09 Problem Solutions

### **Problem 1: Tipler 3-49**

In Compton scattering, the initial state has a photon (energy  $E_1$ ) and an electron at rest, and the final state has another photon (energy  $E_2$ ) and an electron in motion. Conservation of energy says

$$E_1 + mc^2 = E_2 + mc^2 + E_k. \quad (1)$$

Therefore

$$E_k = E_1 - E_2 = h(f_1 - f_2). \quad (2)$$

The Compton effect formula gives

$$\lambda_2 - \lambda_1 = \frac{h}{mc}(1 - \cos \theta). \quad (3)$$

In Eq. (3) rewrite the wavelength in terms of the frequency,

$$\frac{1}{f_2} - \frac{1}{f_1} = \frac{h}{mc^2}(1 - \cos \theta) \quad \Rightarrow \quad \frac{1}{f_2} = \frac{1}{f_1} + \frac{h}{mc^2}(1 - \cos \theta) \quad (4)$$

Solve Eq. (4) for  $f_2$ .

$$f_2 = \frac{mc^2 f_1}{mc^2 + hf_1(1 - \cos \theta)}. \quad (5)$$

Substitute Eq. (5) into Eq. (2).

$$\begin{aligned} E_k &= h \left( f_1 - \frac{mc^2 f_1}{mc^2 + hf_1(1 - \cos \theta)} \right) = hf_1 \left( 1 - \frac{mc^2}{mc^2 + hf_1(1 - \cos \theta)} \right) \\ &= hf_1 \left( \frac{hf_1(1 - \cos \theta)}{mc^2 + hf_1(1 - \cos \theta)} \right) = hf_1 \frac{1}{1 + \frac{mc^2}{hf_1(1 - \cos \theta)}}. \end{aligned} \quad (6)$$

(The last step divided both numerator and denominator by the quantity in the numerator.) This gives the kinetic energy of the electron when the photon scatters at an arbitrary

angle  $\theta$ . The problem asks what is the largest possible value of  $E_k$ . For a fixed initial frequency  $f_1$ ,  $E_k$  is largest when the denominator is smallest. The denominator is smallest when the denominator of the second term is largest. The largest value of  $1 - \cos \theta$  is 2 and that occurs when  $\theta = \pi$  radians (where  $\cos \theta$  is  $-1$ ). Therefore the maximum kinetic energy transferred to the electron is given by

$$E_k = \frac{hf_1}{1 + \frac{mc^2}{2hf_1}}. \quad (7)$$

### **Problem 2: Tipler 4-1**

$$\frac{1}{\lambda_{mn}} = R \left( \frac{1}{m^2} - \frac{1}{n^2} \right) \quad \text{where } R = 1.097 \times 10^7 m^{-1} \quad (\text{Equation 4-2})$$

The Lyman series ends on  $m = 1$ , the Balmer series on  $m = 2$ , and the Paschen series on

$$m = 3. \quad \text{The series limits all have } n = \infty, \text{ so } \frac{1}{n} = 0$$

$$\frac{1}{\lambda_L} = R \left( \frac{1}{1^2} \right) = 1.097 \times 10^7 m^{-1}$$

$$\lambda_L(\text{limit}) = 1.097 \times 10^7 m^{-1} = 91.16 \times 10^{-9} m = 91.16 \text{ nm}$$

$$\frac{1}{\lambda_B} = R \left( \frac{1}{2^2} \right) = 1.097 \times 10^7 m^{-1} / 4$$

$$\lambda_B(\text{limit}) = 4 / 1.097 \times 10^7 m^{-1} = 3.646 \times 10^{-7} m = 364.6 \text{ nm}$$

$$\frac{1}{\lambda_P} = R \left( \frac{1}{3^2} \right) = 1.097 \times 10^7 m^{-1} / 9$$

$$\lambda_P(\text{limit}) = 9 / 1.097 \times 10^7 m^{-1} = 8.204 \times 10^{-7} m = 820.4 \text{ nm}$$

### **Problem 3: Tipler 4-3**

The lines in the Lyman series are given by Eq. (4-2) [Lyman series is described in Example 4-1]

$$\frac{1}{\lambda} = R \left( \frac{1}{1^2} - \frac{1}{n^2} \right), \quad n = 2, 3, 4, 5, \dots \quad (8)$$

Therefore

$$\frac{1}{164.1 \text{ nm}} = \frac{1.0997 \times 10^7 \text{ m}^{-1}}{1 \times 10^9 \frac{\text{nm}}{\text{m}}} \left( 1 - \frac{1}{n^2} \right) = 1.0997 \times 10^{-2} \text{ nm}^{-1} \left( 1 - \frac{1}{n^2} \right), \quad (9)$$

or

$$\frac{1}{n^2} = 1 - \frac{1}{(164.1)(1.0997 \times 10^{-2})} = 1 - 0.5541 = 0.4495. \quad (10)$$

The value of  $n$  is

$$n = \sqrt{\frac{1}{0.4495}} = 1.5 \quad (11)$$

Since this is not an integer, the answer is NO, this is not a line in the Lyman series.

#### **Problem 4: Tipler 4-5**

None of these lines are in the Paschen series, whose limit is 820.4 nm (see Problem 4-1)

and whose first line is given by:  $\frac{1}{\lambda_{34}} = R \left( \frac{1}{3^2} - \frac{1}{4^2} \right) \rightarrow \lambda_{34} = 1875 \text{ nm}$ . Also, none are

in the Brackett series, whose longest wavelength line is 4052 nm (see Problem 4-4).

The Pfund series has  $m = 5$ . Its first three (i.e., longest wavelength) lines have  $n = 6, 7$ , and 8.

$$\frac{1}{\lambda_{56}} = R \left( \frac{1}{5^2} - \frac{1}{6^2} \right) = 1.341 \times 10^5 \text{ m}^{-1}$$

$$\lambda_{56} = 1/1.341 \times 10^5 \text{ m}^{-1} = 7.458 \times 10^{-6} \text{ m} = 7458 \text{ nm}. \text{ Similarly,}$$

$$\lambda_{57} = 1/2.155 \times 10^5 \text{ m}^{-1} = 4.653 \times 10^{-6} \text{ m} = 4653 \text{ nm}$$

$$\lambda_{58} = 1/2.674 \times 10^5 \text{ m}^{-1} = 3.740 \times 10^{-6} \text{ m} = 3740 \text{ nm}$$

Thus, the line at 4103 nm is not a hydrogen spectral line.