

Nov. 1, 2006

KDB

① Tipler 6.3

$$(a) \psi' = \frac{d\psi}{dx} = -\frac{x}{L^2} \psi$$

$$\psi'' = \frac{d^2\psi}{dx^2} = \left[\left(-\frac{x}{L^2} \right) \left(-\frac{x}{L^2} \right) - \frac{1}{L^2} \right] \psi = \left(\frac{x^2}{L^4} - \frac{1}{L^2} \right) \psi$$

Substitute this into the kinetic energy term in the

TISE:

$$-\frac{\hbar^2}{2m} \psi'' + V\psi = E\psi \Rightarrow$$

$$-\frac{\hbar^2}{2m} \left(\frac{x^2}{L^4} - \frac{1}{L^2} \right) \psi + V\psi = E\psi$$

Solve for $V(x)$

$$V(x) = E + \frac{\hbar^2}{2m} \left(\frac{x^2}{L^4} - \frac{1}{L^2} \right)$$

$$= C + \frac{1}{2} k x^2$$

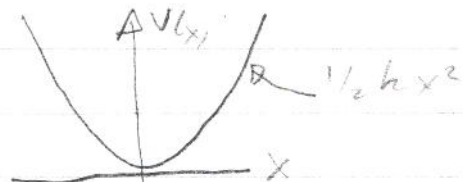
$$k = \frac{\hbar^2}{mL^4}$$

$$C = E - \frac{\hbar^2}{2mL^2} = \frac{\hbar^2}{2mL^2} - \frac{\hbar^2}{2mL^2}$$

$$= 0$$

potential of a harmonic oscillator

$$\boxed{V(x) = \frac{1}{2} k x^2}$$



Problem 2

$$a) \text{ Prob} = \int_{1.0}^{1.1} P(x) dx = \frac{\pi}{2} \int_{1.0}^{1.1} \sin^2 \pi x dx$$

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

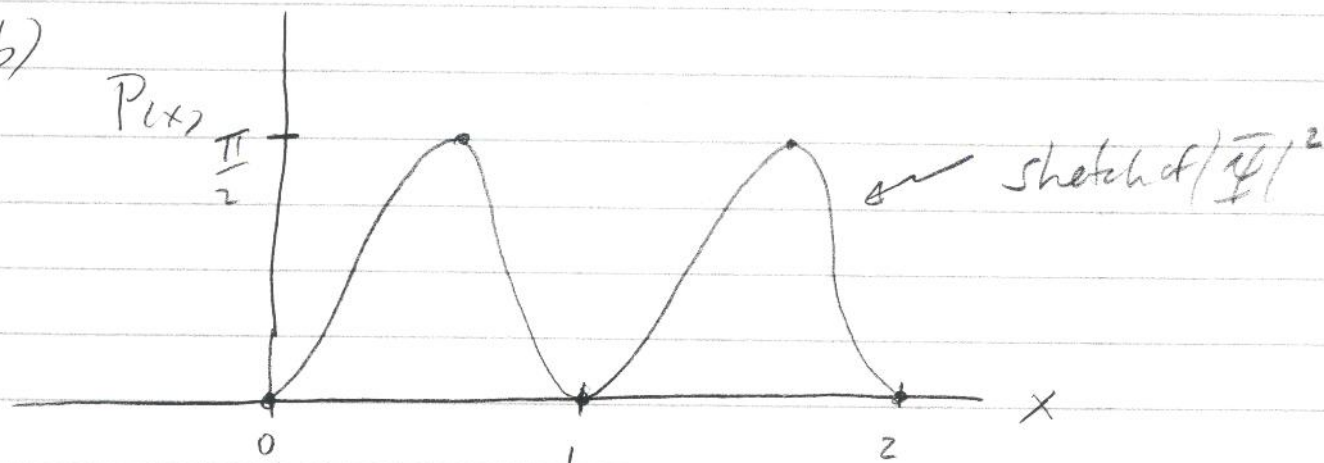
$$\begin{aligned} \therefore \text{ Prob} &= \frac{\pi}{2} \left(\frac{x}{2} - \frac{\sin 2\pi x}{4\pi} \right) \Big|_{1.0}^{1.1} \\ &= \frac{\pi}{2} \left(\frac{0.1}{2} - \frac{\sin 2.2\pi}{4\pi} \right) = \frac{\pi}{40} - \frac{1}{8} \sin 2.2\pi \\ &= 0.0785 - 0.0468 = 0.0317 \end{aligned}$$

Approximation:

$$\begin{aligned} \text{Prob} &= \int_a^b f(x) dx \approx f\left(\frac{b+a}{2}\right) \Delta x = f\left(\frac{b+a}{2}\right) (b-a) \\ &= \frac{\pi}{2} \sin^2[(1.05)\pi] (0.1) = 3.84 \times 10^{-3} \end{aligned}$$

In some sense they agree because the probability is almost 0 in both cases. However, they differ by about a factor of 100 (which is a large factor). This disagreement is large because the derivative of the wavefunction is a maximum near this point, so the change in ψ per increment of x is largest here.

b)



$$P(x) = \bar{\psi} \psi = \frac{\pi}{2} \sin^2 \pi x$$