

Nov. 3, 2006

KDB

① Tipler 6.4

$$\begin{aligned} (a) \text{ K.E.} &= E - V(x) = \frac{\hbar^2}{2mL^2} - \frac{\hbar^2}{2mL^4} x^2 \\ &= \frac{\hbar^2}{2mL^2} \left(1 - \frac{x^2}{L^2}\right) \end{aligned}$$

(b) Classical turning point is where $E = V(x)$, i.e. there is no kinetic energy. Thus, if $\text{K.E.} = 0$

$$\Rightarrow 1 - \frac{x^2}{L^2} = 0 \Rightarrow x = \pm L.$$

$$(c) \quad V(x) = \frac{1}{2} m \omega^2 x^2 \quad (\text{Harmonic oscillator Potential Energy!})$$

$$\therefore \frac{\hbar^2 x^2}{2mL^4} = \frac{m \omega^2 x^2}{2} \quad \therefore \omega^2 = \frac{\hbar^2}{m^2 L^4} \Rightarrow \omega = \frac{\hbar}{mL^2}.$$

$$\text{Thus } E = \frac{\hbar^2}{2mL^2} = \left(\frac{\hbar}{mL^2}\right) \frac{\hbar}{2} = \frac{\hbar \omega}{2}.$$

② 6.6 For a free electron $V(x) = 0$ so the TISE

gives
$$-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \psi = E \psi$$

Now $\psi(x) = A \sin(2.5 \times 10^{10} x)$

$$\frac{d^2}{dx^2} \psi(x) = (2.5 \times 10^{10})^2 \psi(x)$$

Using this in the TISE gives

$$(2.5 \times 10^{10})^2 \frac{\hbar^2}{2m} \psi = E \psi$$

+ for a free particle $E = \text{Kinetic Energy} = \frac{p^2}{2m}$ so

$$p^2 = 2m E = 2m (2.5 \times 10^{10})^2 \frac{\hbar^2}{2m} = (2.5 \times 10^{10} \hbar)^2$$

$$\therefore p = 2.5 \times 10^{10} \hbar = 2.5 \times 10^{10} \frac{6.63 \times 10^{-34}}{2\pi}$$

$$= 2.64 \times 10^{-24} \text{ kg m/s}$$

$$(b) E = \frac{p^2}{2m} = \frac{(2.64 \times 10^{-24} \text{ kg m/s})^2}{2(9.11 \times 10^{-31} \text{ kg})} = 3.82 \times 10^{-18} \text{ J}$$

$$= \frac{3.82 \times 10^{-18} \text{ J}}{1.60 \times 10^{-19} \text{ J/eV}} = 23.9 \text{ eV}$$

$$(c) \quad \lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34} \text{ Js}}{2.64 \times 10^{-24} \text{ kg m/s}} = 2.5 \times 10^{-10} \text{ m} = 0.25 \text{ nm}$$

③ 6.8 Normalization is just a statement that uses probability:

$$\int \bar{\Psi}^* \Psi dx = 1$$

$$\therefore \int_{-a}^a A^2 e^{-i(\hbar x - \omega t)} e^{i(\hbar x - \omega t)} dx = 1$$

$$= A^2 \int_{-a}^a dx = A^2 2a = 1$$

$$\Rightarrow A = \frac{1}{\sqrt{2a}}$$

Obviously if $a \rightarrow \infty$ then $A \rightarrow 0$ so normalization is not valid here. Such a case can be handled using limit procedures that are labelled box normalization.

Tipler p 249, Question 2

NO. For example we know from Example 6.1 on p. 250 that for a free particle the wavefunction

$$\psi(x) = A \sinh x + B \cosh x$$

is a solution to the TISE for any value of $A + B$.

Thus take $A = i$ $B = 1 \Rightarrow$

$$\psi(x) = \cosh x + i \sinh x = e^{ikx}$$

which is not real + is a solution to the TISE.