

2006-11-06 Problem Solutions

Problem 1: Tipler 6-11

The second excited state wave function is ($n = 3$) $\psi_3(x) = \sqrt{2/L} \sin(3\pi x/L)$

(Equation 6-32). The probability of finding the particle in Δx is approximately:

$$P(x)\Delta x = \frac{2}{L} \sin^2\left(\frac{3\pi x}{L}\right) \Delta x$$

$$(a) \ x = \frac{L}{2} \text{ and } \Delta x = 0.002L, \ P(x)\Delta x = \frac{2(0.002L)}{L} \sin^2\left(\frac{3\pi L}{2L}\right) = 0.004 \sin^2 \frac{3\pi}{2} = 0.004$$

$$(b) \ x = \frac{2L}{3}, \ P(x)\Delta x = 0.004 \sin^2\left(\frac{6\pi L}{3L}\right) = 0.004 \sin^2 2\pi = 0$$

$$(c) \ \text{for } x = L, \ P(x)\Delta x = 0.004 \sin^2\left(\frac{3\pi L}{L}\right) = 0.004 \sin^2 3\pi = 0$$

Problem 2: Tipler 6-17

This is an infinite square well with $L = 10 \text{ cm}$.

$$E_n = \frac{h^2 n^2}{8mL^2} = \frac{1}{2} m v^2 = \frac{(2.0 \times 10^{-3} \text{ kg})(20 \text{ nm})^2}{2(3.16 \times 10^7 \text{ s})^2}$$

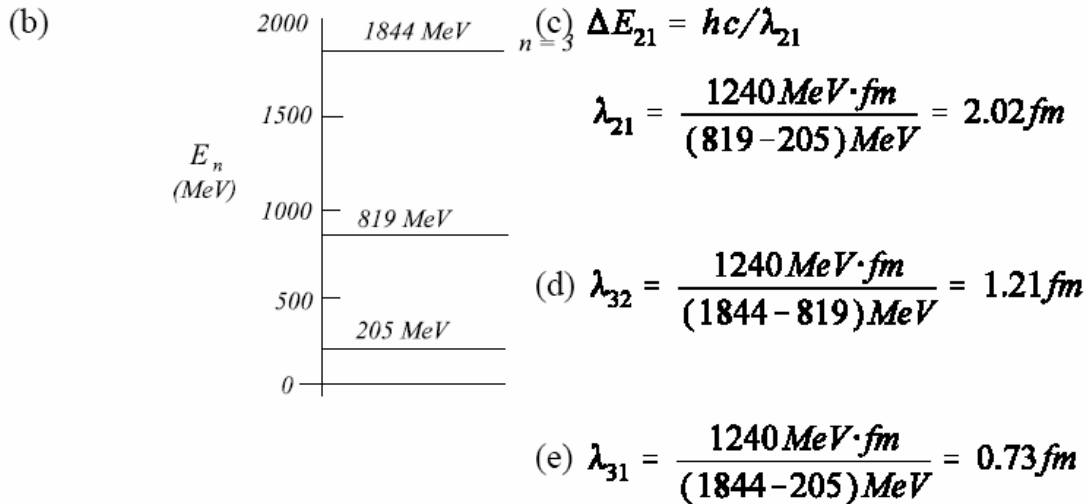
$$n^2 = \frac{8(2.0 \times 10^{-3} \text{ kg})^2 (20 \times 10^{-9} \text{ m})^2 (0.1 \text{ m})^2}{2(3.16 \times 10^7 \text{ s})^2 (6.63 \times 10^{-34} \text{ J}\cdot\text{s})^2}$$

$$n = \frac{2(2.0 \times 10^{-3} \text{ kg})(20 \times 10^{-9} \text{ m})(0.1 \text{ m})}{3.16 \times 10^7 \text{ s}(6.63 \times 10^{-34} \text{ J}\cdot\text{s})} = 3.8 \times 10^{14}$$

Problem 3: Tipler 6-48

(a) $E_n = \frac{n^2 h^2}{8mL^2}$ The ground state is $n = 1$, so

$$E_1 = \frac{(hc)^2}{8(mc^2)L^2} = \frac{(1240 \text{ MeV} \cdot \text{fm})^2}{8(938.3 \text{ MeV})(1 \text{ fm})^2} = 204.8 \text{ MeV}$$



Problem 4: Tipler 6-49

(a) The probability density for the ground state is $P(x) = \psi^2(x) = (2/L)\sin^2 \pi x/L$. The probability of finding the particle in the range $0 < x < L/2$ is :

$$P = \int_0^{L/2} P(x) dx = \frac{2}{L} \frac{L}{\pi} \int_0^{\pi/2} \sin^2 u du = \frac{2}{\pi} \left(\frac{\pi}{4} - 0 \right) = \frac{1}{2} \quad \text{where } u = \pi x/L$$

(b) $P = \int_0^{L/3} P(x) dx = \frac{2}{L} \frac{L}{\pi} \int_0^{\pi/3} \sin^2 u du = \frac{2}{\pi} \left(\frac{\pi}{6} - \frac{\sin 2\pi/3}{4} \right) = \frac{1}{3} - \frac{\sqrt{3}}{4\pi} = 0.195$

(Note $1/3$ is the classical result.)

(c) $P = \int_0^{3L/4} P(x) dx = \frac{2}{L} \frac{L}{\pi} \int_0^{3\pi/4} \sin^2 u du = \frac{2}{\pi} \left(\frac{3\pi}{8} - \frac{\sin 3\pi/2}{4} \right) = \frac{3}{4} + \frac{1}{2\pi} = 0.909$

(Note $3/4$ is the classical result.)