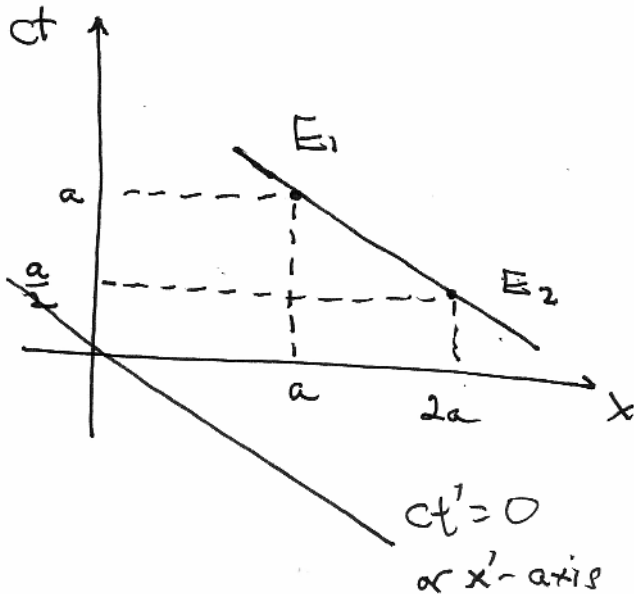


Test 1 Answers

1. a) The Lorentz transformation equations are Eq. (1-20) in the book.
 b) The derivation of the velocity transformation equations is on p. 25 of the book.
2. a)



- b) The spacetime interval is $(c\Delta t)^2 - (\Delta x)^2 = \left(-\frac{a}{2}\right)^2 - a^2 = -\frac{3}{4}a^2$. Since this is negative, the space term dominates, and this is a *spacelike* interval.
- c) The spacetime interval is invariant, so a Lorentz transformation cannot change its value, or its sign. But a transformation can be found that removes the term with the “other” sign from the overall invariant. In this case, since the invariant is negative, the positive term or the time interval can be removed, so that $\Delta t' = 0$. The Lorentz transformation equation to this new frame for the time variable is $t' = \gamma\left(t - \frac{v}{c^2}x\right)$. Apply that to each of the two events 2 and 1 and take the difference, to obtain $\Delta t' = \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right)$. $\Delta t' = 0$ requires $\Delta t - \frac{v}{c^2}\Delta x = 0$.

Calculate these differences for the two events and get $-\frac{a}{2} - \frac{v}{c} \cdot a = 0$, which gives

$\frac{v}{c} = -\frac{1}{2}$. The minus sign means that frame S' goes left.

d) $\gamma = \frac{1}{\sqrt{1 - \left(-\frac{1}{2}\right)^2}} = \frac{1}{\sqrt{\frac{3}{4}}} = \frac{2}{\sqrt{3}} = 1.155$. The Lorentz transformation for position,

applied to the two events and take the difference gives

$$\Delta x' = \gamma(\Delta x - v\Delta t) = \frac{2}{\sqrt{3}} \left[a - \left(-\frac{1}{2}c\right) \left(-\frac{a}{2c}\right) \right] = \frac{2}{\sqrt{3}} a \left(1 - \frac{1}{4} \right) = \frac{\sqrt{3}}{2} a = 0.866a.$$

This can also be calculated from the invariant spacetime interval; with the result from (b) it gives $-(\Delta x')^2 = -\frac{3}{4}a^2$ and gets the same result.

- e) The line drawn through the two events E_1 and E_2 is a line of constant ct' , because we made $\Delta t' = 0$ on it. The line through the origin and parallel to the line joining the two events on the spacetime diagram is the line $ct' = 0$. Therefore it is the x' -axis.

3. The galaxy is receding from the Earth, so we need the Doppler shift formula which gives a lower frequency:

$$f = f_0 \sqrt{\frac{1 - \beta}{1 + \beta}}.$$

The data given in the problem is in terms of wavelengths, so we rewrite the preceding formula using the relation $f = c / \lambda$:

$$\frac{c}{\lambda} = \frac{c}{\lambda_0} \sqrt{\frac{1 - \beta}{1 + \beta}}, \quad \text{or} \quad \lambda = \lambda_0 \sqrt{\frac{1 + \beta}{1 - \beta}}, \quad \text{or} \quad \frac{\lambda}{\lambda_0} = \sqrt{\frac{1 + \beta}{1 - \beta}}.$$

Do some algebra.

$$\left(\frac{\lambda}{\lambda_0}\right)^2 = \frac{1+\beta}{1-\beta} \Rightarrow (1-\beta)\left(\frac{\lambda}{\lambda_0}\right)^2 = 1+\beta \Rightarrow \left(\frac{\lambda}{\lambda_0}\right)^2 - 1 = \left[\left(\frac{\lambda}{\lambda_0}\right)^2 + 1\right]\beta$$

$$\beta = \frac{\left(\frac{\lambda}{\lambda_0}\right)^2 - 1}{\left(\frac{\lambda}{\lambda_0}\right)^2 + 1}$$

Put in the numbers.

$$\beta = \frac{\left(\frac{600}{434}\right)^2 - 1}{\left(\frac{600}{434}\right)^2 + 1}$$

$$\beta = 0.313.$$

4. For $\beta = 0.9$, $\gamma = 2.29$.

- a) The proper mean lifetime is measured in the rest frame of the pions and is a proper time interval. The lifetime measured in the laboratory is longer by the time dilation factor,

$$t = \gamma\tau = 2.29 * (2.6 \times 10^{-8} \text{ s}) = 5.96 \times 10^{-8} \text{ s}.$$

- b) The spacetime interval is invariant and can be calculated in either the rest frame of the pion or the lab frame. In this case, it is easier to do it in the frame of the pion, because in that frame $\Delta x = 0$, and only one calculation is required.

$$(\Delta s)^2 = c^2 (\Delta t_p)^2 = (c\tau)^2 = \left[(3 \times 10^8 \text{ m/s})(2.6 \times 10^{-8} \text{ s})\right]^2 = 60.84 \text{ m}^2,$$

$$\Delta s = 7.8 \text{ m}.$$

It's also correct to do the calculation in the lab frame. Get the Δx value by multiplying the speed by the lifetime in the lab frame calculated in (a). The Δt value is the lifetime calculated in the lab [from (a)]. The answer is the same.