

Here are answers to the 3 problems on the test.

1. The energy of this particle is its rest mass energy m_0c^2 plus its kinetic energy, which is given to be twice the rest mass energy; so the total initial energy is $E_i = 3m_0c^2$.

a) Using the relation between energy and momentum, $E = \sqrt{(m_0c^2)^2 + (pc)^2}$, the initial energy of this particle, with initial momentum p_i , is

$$\begin{aligned}E_i &= 3m_0c^2 = \sqrt{(m_0c^2)^2 + (p_i c)^2}, \\(m_0c^2)^2 + (p_i c)^2 &= 9(m_0c^2)^2, \\(p_i c)^2 &= 8(m_0c^2)^2, \\p_i &= 2\sqrt{2}m_0c.\end{aligned}$$

b) The particle in part (a) makes a totally inelastic collision with a particle of mass $2m_0$, which is initially at rest. The single composite object after the collision has some mass M , and some final momentum p_f . By momentum conservation, the final momentum equals the initial momentum found in (a),

$$p_i = p_f.$$

The total energy of the system is the energy of the moving particle, $3m_0c^2$, plus the rest mass energy of the stationary particle, $2m_0c^2$, so the total energy is $E_{\text{total}} = 5m_0c^2$. This is conserved in the collision so

$$\sqrt{(Mc^2)^2 + (p_f c)^2} = \sqrt{(Mc^2)^2 + (p_i c)^2} = 5(m_0c^2).$$

Substitute the value of p_i from (a) and solve for M .

$$\begin{aligned}\sqrt{(Mc^2)^2 + 8(m_0c^2)^2} &= 5(m_0c^2), \\(Mc^2)^2 + 8(m_0c^2)^2 &= 25(m_0c^2)^2, \\(Mc^2)^2 &= 17(m_0c^2)^2, \\Mc^2 &= \sqrt{17}m_0c^2, \\M &= \sqrt{17}m_0 = 4.12m_0.\end{aligned}$$

c) There are two ways to get the velocity U of the composite mass M after the collision.

One is to use the energy-velocity relation, $E = \frac{Mc^2}{\sqrt{1-U^2/c^2}}$. Substituting the value of E_{total} from above and the value of M just found gives

$$\frac{\sqrt{17}m_0c^2}{\sqrt{1-U^2/c^2}} = 5m_0c^2,$$

$$\frac{1}{\sqrt{1-U^2/c^2}} = \frac{5}{\sqrt{17}},$$

$$1-U^2/c^2 = \frac{17}{25},$$

$$\frac{U^2}{c^2} = \frac{8}{25},$$

$$\frac{U}{c} = \frac{2\sqrt{2}}{5} = 0.57.$$

The second way is to use the formula

$$\frac{U}{c} = \frac{pc}{E} = \frac{2\sqrt{2}m_0c^2}{5m_0c^2} = \frac{2\sqrt{2}}{5}.$$

2. Use Einstein's equation for the photoelectric effect, $eV_s = hf - \phi = hc/\lambda - \phi$, where V_s is the stopping potential, which is the potential difference that reduces the photocurrent to zero.

(a) The potential difference $V = 0.376$ V is the stopping potential for light of wavelength $\lambda = 546.1$ nm.

$$e(0.376 \text{ V}) = (1240 \text{ eV} \cdot \text{nm}) / (546.1 \text{ nm}) - \phi,$$

$$\phi = \frac{1240}{546.1} \text{ eV} - 0.376 \text{ eV} = 1.89 \text{ eV}.$$

The work function ϕ is a property of the material and is independent of the wavelength of the incident light.

(b) Now that we know the work function ϕ , we apply Einstein's equation again with a different wavelength and calculate the stopping potential.

$$eV_s = hc/\lambda - \phi = \frac{1240 \text{ eV} \cdot \text{nm}}{587.5 \text{ nm}} - 1.89 \text{ eV} = 0.221 \text{ eV},$$

$$V_s = 0.221 \text{ V}.$$

3. For the initial photon with $\lambda = 0.110$ nm, its momentum is

$$p_i = \frac{h}{\lambda} = \frac{hc}{\lambda} \cdot \frac{1}{c} = \frac{1240 \text{ eV} \cdot \text{nm}}{0.110 \text{ nm}} \cdot \frac{1}{c} = 1.13 \times 10^4 \frac{\text{eV}}{c}.$$

From the equation for the change in photon wavelength in the Compton effect, $\Delta\lambda = (h/m_e c)(1 - \cos\theta)$, for this head-on collision with $\theta = \pi$, $\cos\theta = -1$,

$$\Delta\lambda = 2 \frac{h}{m_e c} = 2 \frac{hc}{m_e c^2} = 2 \frac{1240 \text{ eV} \cdot \text{nm}}{0.511 \times 10^6 \text{ eV}} = 4.85 \times 10^{-3} \text{ nm}.$$

The momentum of the scattered photon,

$$p_f = -\frac{h}{\lambda_i + \Delta\lambda} = -\frac{hc}{\lambda_i + \Delta\lambda} \cdot \frac{1}{c} = -\frac{1240 \text{ eV} \cdot \text{nm}}{(0.110 + 0.00485) \text{ nm}} \cdot \frac{1}{c} = -1.080 \times 10^4 \frac{\text{eV}}{c}.$$

The negative sign is because the scattered photon is going in the opposite direction from the incident photon. Since the electron was initially at rest, momentum conservation requires that its final momentum $p_{e,f}$ is the difference of the initial and final photon momenta,

$$p_{e,f} = 2.21 \times 10^4 \frac{\text{eV}}{c} \quad \text{or} \quad p_{e,f} c = 2.21 \times 10^4 \text{ eV}.$$

Since this is small compared to the rest-mass energy of the electron (0.511 MeV), we can use the non-relativistic equation to get the final kinetic energy of the electron,

$$E_{e,k,f} = \frac{p_{e,f}^2}{2m} = \frac{(p_{e,f} c)^2}{2mc^2} = \frac{(2.21 \times 10^4 \text{ eV})^2}{2(0.511 \times 10^6)} = 478 \text{ eV}.$$