

TEST #3

Prob. 1

$$\psi(x) = A\sqrt{x} \quad 0 \leq x \leq 2$$

$$\psi(x) = 0 \quad \text{elsewhere}$$

(a) Normalization: $\int_{-\infty}^{\infty} \psi^* \psi dx = 1$

$$\therefore \int_0^2 A^2 x dx = 1 \Rightarrow A^2 \int_0^2 x dx = A^2 \left. \frac{x^2}{2} \right|_0^2 = A^2 \cdot 2 = 1$$
$$\Rightarrow A = \frac{1}{\sqrt{2}}$$

(b) $\text{Prob}(x_1 \leq x \leq x_2) = \int_{x_1}^{x_2} \psi^* \psi dx = \int_4^{400} 0 dx = 0.$

(c) $\text{Prob}(x_1 \leq x \leq x_2) = \int_{x_1}^{x_2} \psi^* \psi dx = \int_1^2 A^2 x dx = A^2 \left. \frac{x^2}{2} \right|_1^2 = \frac{A^2}{2} (4-1)$
$$= \frac{3A^2}{2} = \frac{3}{4}.$$

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Delta x \geq$$

2. The shortest wavelength has the highest energy

a) $n_i = \infty$, this is 13.6 eV

$$E = \frac{hc}{\lambda} = 13.6 \text{ eV} \rightarrow \lambda = \frac{hc}{E} = \frac{1240 \text{ eV} \cdot \text{nm}}{13.6 \text{ eV}} = 91.2 \text{ nm}$$

b) The energy is proportional to the reduced mass

$$E_{\mu} = \frac{\mu_{\mu}}{\mu_e} E_e$$

$$\mu = \frac{m}{1 + m/M}$$

$$\mu_{\mu} = \frac{105.7 \text{ MeV}/c^2}{1 + 105.7/438}$$

$\hookrightarrow$ Energy of muonic atom

$$\mu_e = \frac{0.511 \text{ MeV}/c^2}{1 + 0.511/438}$$

$$E_{\mu} = 185.4 \cdot 13.6 = 2528 \text{ eV}$$

$$\lambda = \frac{hc}{E} = \frac{1240 \text{ eV} \cdot \text{nm}}{2528 \text{ eV}} = 0.49 \text{ nm}$$

(2)

The radius will be given by

$$r_{\mu} = \frac{\mu_e}{\mu_{\mu}} r_e = \frac{0.0529 \text{ nm}}{185.4} = 2.85 \times 10^{-4} \text{ nm}$$

$$3. a) p = h/\lambda \Rightarrow v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{1 \text{ kg} \cdot 1 \times 10^{-10} \text{ m}} = 6.63 \times 10^{-24} \text{ m/s}$$

$$b) eV_0 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{(hc)^2}{2mc^2\lambda^2}$$

$$V_0 = \frac{1}{e} \frac{(1240 \text{ eV}\cdot\text{nm})^2}{2 (5.11 \times 10^5 \text{ eV}) (0.1 \text{ nm})^2} = 150 \text{ V}$$

$$c) p = h/\lambda = \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{10^{-10} \text{ m}} = 6.63 \times 10^{-24} \text{ kg m/s}$$

$$d) \Delta x \Delta p \geq \hbar/2$$

$$\Delta x \geq \frac{\hbar/2}{\Delta p} = \frac{1}{4\pi} \frac{6.63 \times 10^{-34}}{6.63 \times 10^{-24}} \text{ m} = \frac{10^{-10}}{4\pi} \text{ m}$$

$$= 0.008 \text{ nm}$$

TEST #3

(4) (a) TISE

$$-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \psi(x) + V(x) \psi(x) = E \psi(x)$$

(b) $\psi(x) = e^{ikx}$

$V(x) = 0$

TISE : $-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \psi(x) = E \psi(x)$

$$\frac{d^2}{dx^2} e^{ikx} = \frac{d}{dx} ik e^{ikx} = (ik)^2 e^{ikx} = -k^2 e^{ikx}$$

$$\therefore -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \psi(x) = \frac{\hbar^2 k^2}{2m} \psi(x) = E \psi(x)$$

so $E = \frac{\hbar^2 k^2}{2m}$

(c) $\phi(t) = e^{-iEt/\hbar}$