

$$4-20. \quad E = -Z'^2 E_o / n^2 \quad Z' = \left[\frac{-n^2 E}{E_o} \right]^{1/2} = \left[\frac{-2^2 (-5.39 \text{ eV})}{13.6 \text{ eV}} \right]^{1/2} = 1.26$$

4-24. (a) The reduced mass correction to the Rydberg constant is important in this case.

$$R = R_\infty \left(\frac{1}{1 + m/M} \right) = R_\infty \left(\frac{1}{2} \right) = 5.4869 \times 10^6 \text{ m}^{-1} \quad (\text{from Equation 4-26})$$

$$E_n = -hcR/n^2 \quad (\text{from Equation 4-23})$$

$$E_1 = -(1240 \text{ eV} \cdot \text{nm}) (5.4869 \times 10^6 \text{ m}^{-1}) (10^{-9} \text{ m/nm}) / (1)^2 = -6.804 \text{ eV}$$

$$\text{Similarly, } E_2 = -1.701 \text{ eV} \quad \text{and} \quad E_3 = -0.756 \text{ eV}$$

(b) Lyman α is the $n = 2 \rightarrow n = 1$ transition.

$$\frac{hc}{\lambda_\alpha} = E_2 - E_1 \rightarrow \lambda_\alpha = \frac{hc}{E_2 - E_1} = \frac{1240 \text{ eV} \cdot \text{nm}}{-1.701 \text{ eV} - (-6.804 \text{ eV})} = 243 \text{ nm}$$

Lyman β is the $n = 3 \rightarrow n = 1$ transition.

$$\lambda_\beta = \frac{hc}{E_3 - E_1} = \frac{1240 \text{ eV} \cdot \text{nm}}{-0.756 \text{ eV} - (-6.804 \text{ eV})} = 205 \text{ nm}$$

$$4-41. \quad (a) \quad i = q f_{\text{rev}} = e \frac{Z^2 m k^2 e^4}{2 \pi \hbar^3 n^3} \quad (\text{from Equation 4-28})$$

$$\begin{aligned} &= e \frac{m c^2 (k e^2)^2 (1)^2}{2 \pi \hbar (\hbar c)^2 (1)^3} = \frac{e c}{(h/mc)} \left(\frac{k e^2}{\hbar c} \right)^2 = \frac{e c \alpha^2}{\lambda_c} \\ &= \frac{(1.602 \times 10^{-19} \text{ C})(3.00 \times 10^{17} \text{ nm/s})}{0.00243 \text{ nm}} \left(\frac{1}{137} \right)^2 = 1.054 \times 10^{-3} \text{ A} \end{aligned}$$

$$\begin{aligned} (b) \quad \mu &= i A = i \pi a_o^2 = \left(\frac{e m k^2 e^4}{2 \pi \hbar^3} \right) \pi \left(\frac{\hbar^2}{m k e^2} \right) = \frac{e \hbar}{2 m} \\ &= \frac{(1.602 \times 10^{-19} \text{ C})(1.055 \times 10^{-34} \text{ J} \cdot \text{s})}{2(9.11 \times 10^{-31} \text{ kg})} = 9.28 \times 10^{-24} \text{ A} \cdot \text{m}^2 \end{aligned}$$

or

$$= (1.054 \times 10^{-3} \text{ A}) \pi (0.529 \times 10^{-10} \text{ m})^2 = 9.27 \times 10^{-24} \text{ A} \cdot \text{m}^2$$

4-56. Centripetal acceleration would be provided by the gravitational force:

$$F_G = G \frac{Mm}{r^2} = \frac{mv^2}{r} \quad M = \text{proton mass and } m = \text{electron mass, so } v = \left(\frac{GM}{r} \right)^{1/2}$$

$$L = mvr = n\hbar \rightarrow r = n\hbar/mv \text{ or}$$

$$r_n = \frac{n\hbar}{m(GM/r_n)^{1/2}} \rightarrow r_n^2 = \frac{n^2\hbar^2 r_n}{m^2 GM} \text{ and, } r_n = \frac{n^2\hbar^2}{GMm^2} \rightarrow a_0 = \frac{\hbar^2}{GMm^2}$$

$$\text{The total energy is: } E = \frac{1}{2}mv^2 + \left(-\frac{GMm}{r} \right) = \frac{1}{2}m \left(\frac{GM}{r} \right) - \frac{GMm}{2r}$$

$$E_n = -\frac{GMm}{2r_n} = -\frac{(GMm)(GMm^2)}{2n^2\hbar^2} = -\frac{G^2M^2m^3}{2n^2\hbar^2}$$

$$\text{The gravitational H}\alpha \text{ line is: } \Delta E = E_2 - E_3 = \frac{G^2M^2m^3}{2\hbar^2} \left(\frac{1}{2^2} - \frac{1}{3^2} \right)$$

$$\Delta E = \frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)^2 (1.67 \times 10^{-27} \text{ kg})^2 (9.11 \times 10^{-31} \text{ kg})^3 (0.1389)}{2(1.055 \times 10^{-34})^2}$$

$$= 5.85 \times 10^{-98} \text{ J} = 3.66 \times 10^{-79} \text{ eV}$$

$$f = \frac{\Delta E}{h} = \frac{5.85 \times 10^{-98} \text{ J}}{6.63 \times 10^{-34} \text{ J}\cdot\text{s}} = 8.28 \times 10^{-65} \text{ Hz}$$